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Safe Reinforcement Learning to Make Decisions in Robotics

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Abstract

Reinforcement Learning (RL) has demonstrated remarkable performance across various domains, achieving notable successes in areas such as the game of Go, autonomous driving, and robotic control. Despite these achievements, the widespread adoption of RL techniques remains largely confined to simulated environments, primarily due to safety concerns associated with their deployment in real-world settings. To address these limitations, safe RL has emerged as a critical area, receiving significant attention, particularly in robotic decision-making. Safe robot RL aims to overcome the challenges of ensuring safety while enhancing robot performance in complex environments. Achieving safe RL necessitates multifaceted approaches that are efficient in ensuring safety, multifunctional in satisfying multi-objective requirements, cooperative in working with other robots, and robust in carrying out tasks in uncertain environments. These approaches are pivotal for the advancement of safe robot RL.

To satisfy the above requirements, we mainly focus on: 1. Balancing Reward and Safety Optimization: This involves developing strategies that help robots efficiently and safely conduct tasks by balancing the reward performance with the necessity of maintaining safety. 2. Safe Multi-Objective RL: Addressing the need to satisfy multiple requirements in robot learning, this aspect of safe RL emphasizes the ability to meet various objectives while ensuring safety. 3. Safe Multi-Robot RL: In scenarios involving complex tasks that require the coordination of multiple robots, the emphasis is on establishing methods that enable safe and effective cooperation among these robots. 4. Safe and Robust Robot RL: Recognizing the uncertainties inherent in real-world environments, we develop methodologies that are not only safe but also robust, ensuring reliable task performance under a range of conditions. 5. Safe robot RL benchmarks: In order to fairly assess the safety and reward performance of safe robot RL, encompassing both single-robot and multi-robot RL paradigms, and to further the advancement of research in this field, we have designed a set of benchmarks tailored explicitly for safe robot RL research. These benchmarks provide a standardized framework for evaluating safe robot RL methods. Their introduction marks a significant step forward for the safe robot RL community, offering valuable resources for researchers to validate and compare the safety and reward performance of various approaches within this domain.

In conclusion, these works can serve as vital cornerstones in advancing safe RL, particularly in transitioning RL from simulated environments to practical, real-world implementations. Moreover, to further facilitate the deployment of RL in real-world settings, we introduce several open challenges and research directions for future research.

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1

Introduction

Ensuring safe interactions between a robot and its environment is a paramount concern when deploying Reinforcement Learning (RL) in robotics. This chapter begins by providing essential background on safe RL, followed by a concise overview of the field. We then explore the challenges inherent in safe RL implementation and introduce our motivation for addressing these issues. The chapter concludes by outlining the specific scope and contributions of this dissertation.

1.1 Background

RL has its roots in animal learning theory [234]. The field has evolved along various trajectories, shaping the landscape of modern RL. At its core, RL embodies the concept of trial-and-error learning, as introduced by Turing, where experiences are molded by the dual forces of pain and pleasure in response to stimuli. This fundamental principle remains central to RL's approach to problem-solving and decision-making.

When a configuration is reached for which the action is undetermined, a random choice for the missing data is made and the appropriate entry is made in the description, tentatively, and is applied. When a pain stimulus occurs all tentative entries are cancelled, and when a pleasure stimulus occurs they are all made permanent. (Turing, 1948) [234, 248]

By leveraging the principle of RL, in recent years, modern RL has been widely adopted in many fields, e.g. transportation schedule [19, 48, 140, 227], traffic signal control [43, 54], energy management [170], wireless security [159], satellite docking [70], edge computing [263], chemical processes [214], video games [27, 136, 178, 196, 233, 265], board games of Go, shogi, chess and arcade game PAC-MAN [112, 224–226], finance [3, 130, 236, 246], autonomous driving [86, 110, 122, 163, 175, 211], recommender systems [50, 221, 291], resource allocation [116, 153, 171], communication and networks [35, 36, 103, 151], smart grids [129, 215], video compression [165, 262], and robotics [4, 14, 31, 40, 89, 95, 106, 126, 139, 174, 187, 197, 200, 209], etc. However, a challenging problem in this domain is: **how do we guarantee safety when we deploy RL in applications?** After all, unacceptable catastrophes may arise if we fail to take safety into account during RL applications in real-world scenarios, especially in robotics [88]. For example, it must not hurt humans when robots interact with humans in human-machine interaction environments; false or racially discriminating information should not be recommended for people in recommender

systems; safety has to be ensured when self-driving cars are carrying out tasks in real-world environments [94]. More specifically, we introduce several safety definitions from different perspectives, which might be helpful for safe RL research.

Safety definition. The first type of safety definition: according to the definition of Oxford dictionary [230], the phrase “safety” is commonly interpreted to mean “the condition of being protected from or unlikely to cause danger, risk, or injury.” The second type of safety definition: the definition of general “safety” according to wiki ¹, the state of being “safe” is defined as “being protected from harm or other dangers”; “controlling recognized dangers to attain an acceptable level of risk” is also referred to as “safety”. The third type of safety definition: according to Hans *et al.* [98], humans need to label environmental states as “safe” or “unsafe,” and agents are considered “safe” if “they never reach unsafe states”. The fourth type of safety definition: agents are considered to be “safe” by some researchers [97, 109, 138] if “they act, reason, and generalize obeying human desires”. The fifth type of safety definitions: Moldovan and Abbeel [180] consider an agent “safe” if “it meets an ergodicity requirement: it can reach each state it visits from any other state it visits, allowing for reversible errors”. In this dissertation, based on the above various definitions, **we define safety as the ability of agents to operate without causing harm to their environments or themselves across diverse real-world settings, where “harm” refers to any physical, psychological, or environmental damage that may adversely affect agents’ environments or themselves.** Furthermore, we investigate safe RL methods, which are about optimizing cost objectives (e.g., Chapters 3, 4, 5, 6, and 7), improving undesirable situations, reducing risk, and controlling agents to be safe, etc. Specifically, if the agent action violates safety constraints, a cost value will be emitted from its interaction environments. Safety in robot RL systems is approached differently during the training and deployment phases. In the training phase, safety is usually defined as the agent’s adherence to simulated constraints, where the agent needs to learn to avoid actions that result in penalties, and a cost function is leveraged to help the agent learn safe actions. In the deployment phase, safety is described by the agent’s performance within real-world constraints limits, and safety control is real-time and potentially involves human operations.

The concept of safe RL has emerged as a crucial area of research in response to the growing need for safety in robot RL applications [94]. This development marks a pivotal shift, focusing not only on achieving high performance but also on ensuring adherence to strict safety standards, especially in complex and unpredictable real-world environments. Ensuring safety in RL poses a significant practical challenge and remains one of the key unresolved issues in AI safety [12]. As a result, it has garnered increasing attention within the AI community, underscoring its importance.

1.2 Overview

Numerous safe RL methods have been proposed over the past few decades [21, 121, 166]. In particular, safe RL approaches for robotics emphasize optimizing robot action and state, as well as modeling dynamics, to ensure safe learning. For instance, Gaussian models are used to model the safe state space [7, 24, 231, 232, 247, 254]; formal methods are leveraged to verify safe action and state space [80, 127, 277]; control theory is applied to search safe action space [51, 52, 128, 142, 168]. Moreover, there are several types of safe RL methods based on different criteria. For example, from the perspective of optimization criteria, several methods consider cost as one of the optimization objectives to achieve safety, e.g., [20, 32, 101, 105, 117, 160, 183, 213, 236]; some methods consider safety in RL exploration process

¹<https://en.wikipedia.org/wiki/Safety>

by leveraging external knowledge, e.g., [1, 51, 57, 83, 180, 238, 243]. From the perspectives of policy and value-based methods for safe RL, summarise as follows, policy-based safe RL: [4, 68, 89, 266, 271, 282], value-based safe RL: [17, 67, 71, 100, 119, 150, 177, 257]. From the perspective of the agent number, we have safe RL methods in a safe single-agent RL setting and a safe multi-agent RL setting [94]. In safe single robot RL, an agent needs to explore the environments to improve its reward while keeping costs below the constraint bounds. In contrast to safe single-agent RL methods, safe multi-agent RL methods not only need to consider the ego agent’s reward and other agent’s reward, but also have to take into account the ego agent’s safety and other agents’ safety in an unstable multi-agent system.

Balancing reward and safety performance is crucial within these settings. Optimizing for reward can impact safety, and vice versa. The inherent conflicts between reward and safety optimization may lead to suboptimal outcomes or compromise overall performance. Careful calibration is necessary to achieve an effective balance between these competing objectives. Various methodologies have been proposed to address this challenge [5, 208, 266, 274]. For instance, derived from TRPO [216], CPO [5] employs a surrogate model with line search to balance reward and safety performance while ensuring safety constraints. Although CPO demonstrates impressive performance by guaranteeing monotonic reward improvement within safety bounds, its computational demands can be substantial due to trust region optimization. Furthermore, PPO-Lag [208] are leveraged to balance the reward and safety performance with Lagrangian multipliers. The method is general and easy to implement, though it relies on primal-dual optimization, which may be sensitive to dual parameters and heavily relies on initialization. Moreover, a primal method, CRPO [266], is proposed to address the limitations of primal-dual methods, which can directly optimize reward performance or safety performance. Their method shows better performance than primal-dual methods. Nonetheless, it does not effectively address the balance between reward and safety performance due to oscillation during policy optimization. Our method, PCRPO [92], is introduced to handle the oscillations between reward and safety policy optimization, where a policy projection is leveraged to balance performance optimization with a soft framework. Our method can significantly improve reward performance while ensuring safety.

Building upon safe RL balance optimization, we extend our focus to safe multi-objective optimization for real-world applications, addressing the multiple requirements inherent in practical settings. However, balancing multi-objective optimization presents significant challenges due to the conflicting objectives, surpassing the difficulties encountered in conventional safe RL methods. The field of safe multi-objective RL remains relatively unexplored, with limited research available. The most comparable work to our approach is the Learning Preferences and Policies in Parallel (LP3) algorithm [107], which builds upon the Multi-Objective Maximum Posterior Policy optimization (MO-MPO) framework [2]. However, LP3 heavily relies on Q-value estimation, which may not accurately represent safe preferences. Moreover, the study does not provide convergence guarantees in their study. Drawing from our primal settings, we propose a novel safe multi-objective RL framework based on conflict-averse natural policy gradient methods. We first analyze the conflict gradient between each objective’s performance. Furthermore, we provide rigorous convergence analysis and safety violation guarantees, leveraging gradient manipulation techniques and constraint rectification mechanisms. This framework advances the field by addressing the complexities of multi-objective optimization while maintaining a strong emphasis on safety constraints, thus bridging the gap between theoretical foundations and practical applications in safe multi-objective RL.

Following multi-objective optimization, it is important to consider safe multi-robot learning. This consideration is crucial because multi-robot collaboration enables the completion of tasks beyond a single robot’s capabilities in real-world scenarios. Multi-robot systems enhance operational efficiency, which can be helpful for addressing complex, distributed tasks where

individual robots may face limitations. Aiming for safe multi-robot policy optimization, based on formal methods [260], Safe MARL via shielding is proposed [72], in which they study the safe MARL problem by one of the formal methods, Linear Temporal Logic (LTL) [201] that is already adopted in safe single-agent settings [8]. Specifically, an attempt is made to ensure the safety of multi-agent systems by leveraging central shielding or factored shielding methods, which are used to specify the safe actions that agents only can take, and also correct the unsafe actions that agents explored. Further, they evaluate the methods based on Multi-Agent Deep Deterministic Policy Gradient (MADDPG) [157] and Coordinating Q-learning (CQ-learning) methods [62]. Although this method can guarantee safety for multi-agent systems to some extent, it requires a prior LTL safety specification and problem-specific knowledge in advance. In a similar way, [72] ensures MARL safety by optimizing the exploration, [222] developed a safe MARL algorithm by adding a safety layer based on safe DDPG [60] and MADDPG [157] for continuous action space. As a result, their method greatly enhanced safety on some MARL tasks. However, the method does not guarantee zero violation in all tasks that they carry out, and their methods may not guarantee reward monotonic performance improvement while ensuring safety. To address the above limitations, we present a safe MARL framework [89] to guarantee reward monotonic improvement while ensuring safety, in which several algorithms are developed based on the framework, such as MACPO and MAPPO-L algorithms. Derived from the framework, we deploy safe MARL for real-world robot control with soft policy optimization [87], which shows better performance than the existing baselines.

While the aforementioned methods demonstrate notable performance in their respective tasks, achieving zero safety violations, particularly during the training phase, remains a significant challenge. To address this issue, several approaches have been developed. For instance, Anayo *et al.* [7] introduced a reachability-based method using Gaussian models, enabling the agent to adaptively learn unknown system dynamics and identify the maximal safe set. Similarly, Berkenkamp *et al.* [24] proposed a region of attraction method that employs Gaussian processes and Bayesian optimization to guarantee a safe state space. While this method provides a theoretical framework for safe robot learning, its assumptions may be too strong for practical applications. Beyond Gaussian methods, recent studies have explored the use of formal methods in safe robot RL. Nathan and Andre [80] implemented formal verification to check the correctness of the state transition model and select safe action during policy learning. Additionally, control theory-based approaches have been proposed to enhance learning safety. For example, Chow *et al.* [51, 52] utilized Lyapunov functions, based on discrete and continuous control methods, to ensure the global safety of behavior policies in RL. However, designing Lyapunov functions tailored to different environments can be challenging, and the uncertain factors may need to be further considered in their methods. To address the above safety concerns, even in uncertain environments, we have developed a robust, safe RL method that operates on a constrained manifold. In our approach, we construct a Tangent space to facilitate the search for safe policies using projection techniques on constrained manifolds. Experimental results indicate that our method outperforms strong baselines and ensures near-zero safety violations, a level of safety assurance that baselines cannot consistently guarantee.

Finally, to facilitate a fair evaluation of safe RL methods, we have developed a comprehensive suite of benchmarks tailored for the safe RL community. These benchmarks encompass a range of scenarios, including a single-agent safe RL benchmark, a safe multi-objective RL benchmark, and safe multi-agent RL benchmarks. Each benchmark is designed to rigorously test different safety and performance factors across various safe RL methods, providing a standardized platform for assessing the effectiveness of new and existing algorithms.

Summary: In conclusion, this section offers a concise overview of safe RL algorithms, analysing them from various angles, including safe single-agent RL, safe multi-agent RL. While



Figure 1.1: Robots work with humans at an industrial factory.

numerous algorithms have demonstrated remarkable performance in achieving high reward scores, the path to their application in real-world scenarios remains open. It is particularly true in addressing real-world demands, such as balancing reward and safety optimization, managing multi-objective optimization, and coordinating multi-robot policy optimization challenges. Achieving safe robot learning with zero violations is paramount, especially in uncertain environments. To advance the field of safe robot RL, it is essential to develop comprehensive benchmarks. These benchmarks should be designed to impartially evaluate the performance of safe robot RL methods, thereby contributing to a deeper understanding and refinement of these techniques in real-world applications.

1.3 Motivation

In this dissertation, the primary objective is to develop safe learning methods for decision-making in robotics. The critical importance of ensuring safety in robot RL cannot be overstated, particularly in light of the growing deployment of robotic systems across various complex and dynamic settings. This imperative is driven by the potential risks and ethical implications inherent in the actions and interactions of robots within real-world environments. Accordingly, the focus of this research is to establish and refine safe RL methodologies that not only enhance the decision-making prowess of robotic systems but also integrate safety as a core component of their operational framework. This work introduces and elaborates on various safe RL methods and benchmarks, designed to assess and assure safety in both theoretical and practical dimensions of robot learning. These contributions are helpful in fostering trust and reliability in robotic systems, particularly in scenarios marked by prevalent human-robot interactions (As shown in Figure 1.1²).

²Images generated by ChatGPT. Retrieval date: November 14, 2023.

1.4 Challenges

In some applications, the agent’s safety is much more important than the agent’s reward [69], especially in robotics. In this dissertation, five pivotal challenges have been identified to ensure the safety of robot learning and facilitate the advancement of RL in robotic applications. These challenges are detailed as the follows. This delineation serves as a foundational framework for addressing critical areas of concern and sets the stage for further exploration and resolution within the area.

- (1) **Safety Balance.** How can we balance reward and safety performance during policy optimization?
- (2) **Safety Multi-Objective RL.** How can we manage multi-objective policy optimization while ensuring learning safety?
- (3) **Safety Multi-Robot RL.** How can we ensure multi-robot safety while guaranteeing monotonic reward improvement?
- (4) **Zero Violation.** How can we achieve zero violation for safe robot learning during uncertain disturbances?
- (5) **Safety Benchmarks.** What benchmarks can we provide for safe RL communities to fairly and holistically examine safe RL performance?

Ensuring the safety of robot learning while achieving monotonic improvements in RL reward requires a delicate balance between optimizing reward and safety performance. The core challenge lies in managing the trade-off between maximizing rewards and maintaining safety during exploration. It is crucial to acknowledge that policy optimization aimed at improving rewards may inadvertently compromise safety. Therefore, the primary challenge is to strike a balance between optimizing policies for both reward and safety.

Subsequent to addressing the primary challenge of balancing reward and safety policy optimization, a second challenge emerges, centered on effectively managing multiple objectives while adhering to safety constraints. For instance, in industrial settings, robots are often required to fulfill multiple objectives and execute various tasks while concurrently ensuring the safety of the robotic system and its surrounding environment. This necessitates an approach to handling diverse requirements and safety considerations.

After addressing the challenge of safe multi-objective RL, a significant dilemma within the field of robotics is around the cooperative and safe control of multiple robots in real-world applications. Remarkably, the development of control methodologies for multiple robots, grounded in the paradigm of safe multi-robot RL, still needs to be explored. Following this challenge, a fundamental question arises: Can near-zero violations be achieved in robot learning within uncertain environments?

Real-world scenarios often involve inherent uncertainties, background noise, and operation within high-dimensional state spaces. The complexity of these conditions highlights the challenges and urgency of ensuring safety in robot learning, especially when applying RL techniques in practical, real-world environments.

A key challenge in the field is the development of comprehensive benchmarks for safe RL, which are crucial for advancing the safe RL community. Establishing these benchmarks requires ensuring impartial evaluation of algorithms and meaningful contributions to the progress of safe robot learning. This involves defining clear criteria and metrics that accurately evaluate RL algorithms’ reward performance and safety aspects, fostering an environment for rigorous testing and meaningful comparisons. Developing such benchmarks is essential

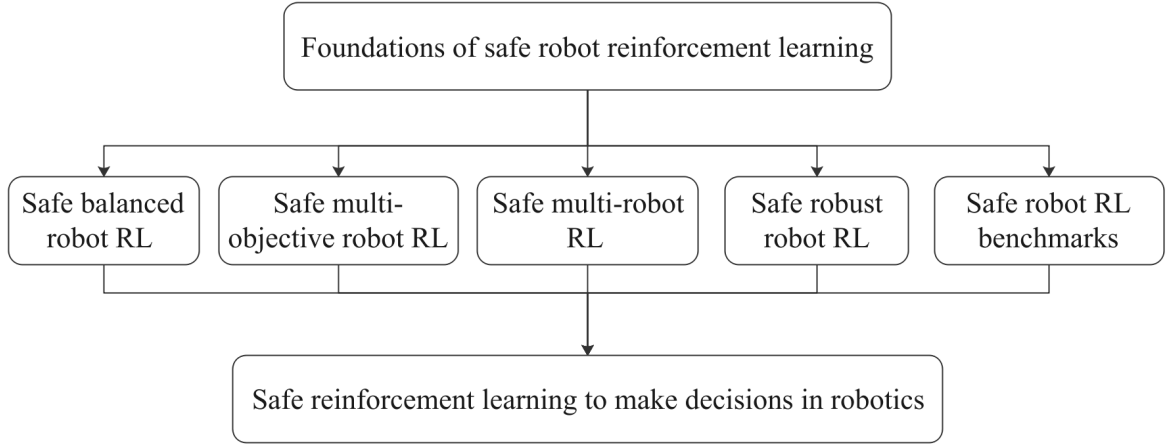


Figure 1.2: An overview of safe robot reinforcement learning

for setting industry standards, guiding research directions, and enabling collaborative efforts toward improving the safety and effectiveness of RL applications in robotics.

1.5 Scope and Contributions

As delineated in Figure 1.2, this dissertation primarily focuses on investigating the identified five challenges and formulating corresponding methods to address them. This exploration is integral to advancing the field by providing structured solutions to these key areas of concern.

To address the first challenge (**Safety Balance**), we aim to handle this conflicting relation by leveraging the theory of gradient manipulation. Initially, we analyze the conflict between reward and safety gradients. Subsequently, we tackle the balance between reward and safety optimization by proposing a soft switching policy optimization method, for which we provide convergence analysis. Based on our theoretical examination, we provide a safe RL framework to overcome the aforementioned challenge. Finally, we evaluate the effectiveness of our method on the *Safety-MuJoCo Benchmark* and a popular safe benchmark, *Omnisafe*. Experimental results demonstrate that our algorithms outperform several state-of-the-art (SOTA) baselines in terms of balancing reward and safety optimization.

To tackle the second challenge (**Safety Multi-Objective RL**), we propose a primal-based framework that handle policy optimization between multi-objective learning and constraint adherence. Our method employs a novel natural policy gradient manipulation method to optimize multiple RL objectives and overcome conflicting gradients between different objectives, our method can perform better than simple weighted average gradient methods, these methods may not be beneficial for specific objectives due to misaligned gradients of different objectives. For instance, during multi-objective reward optimization, if a hard constraint is violated, our algorithm intervenes to adjust the policy and minimize the violation. Particularly, We establish theoretical convergence and constraint violation guarantees, and our proposed method also outperforms prior SOTA methods on challenging safe multi-objective RL tasks.

As for the third challenge, to ensure multi-robot RL safety (**Safety Multi-Robot RL**), in this work, we investigate safe MARL for multi-robot control on cooperative tasks, in which each individual robot has to not only meet its own safety constraints while maximising their reward, but also consider those of others to guarantee safe team behaviours. Firstly, we formulate the safe MARL problem as a constrained Markov game and employ policy optimisation to

solve it theoretically. The proposed algorithm guarantees monotonic improvement in reward and satisfaction of safety constraints. Secondly, as approximations to the theoretical solution, we propose two safe multi-agent policy gradient methods: *Multi-Agent Constrained Policy Optimisation* (MACPO) and *MAPPO-Lagrangian*. Finally, experimental results on the three safe MARL benchmarks indicate that our methods can achieve SOTA performance in the balance between improving reward and satisfying safety constraints compared with strong baselines.

For the fourth challenge, **Zero Violation**, real-world applications are often characterized by inherent uncertainties, noise, and high-dimensional state spaces. This complexity heightens the urgency and difficulty of guaranteeing zero violation in robot learning, particularly when applying RL techniques in practical environments. In this work, we develop a robust and safe RL methodology based on manifolds. We begin by constructing constrained manifolds that incorporate safety constraints. Building on this foundation, we propose a robust safe RL approach, supported by theoretical analysis, using Value-at-Risk (VaR) and Conditional Value-at-Risk (CVaR) to enhance safety robustness. Our methodology is specifically designed to ensure safety within stochastic constraint environments. Following the theoretical analysis, we introduce a practical algorithm—ROSCOM (Robust Safe Policy Optimization on Stochastic Constraint Manifolds)—to search for safe policies on these manifolds. We evaluate the effectiveness of ROSCOM through experiments on circular motion and air-hockey tasks, demonstrating that it outperforms existing baselines in both reward and safety performance.

Specifically, compared to SOTA baselines, our method provides remarkable performance in both safety and reward metrics. For instance, in the air-hockey robot learning task, our approach achieves a 50% improvement in safety performance over the ATACOM framework [149], while also delivering superior rewards. Furthermore, compared to traditional algorithms—including CPO [5], PCPO [271], FOCOPS [290], and PPO-Lagrangian [208]—our method achieves a 99% improvement in safety performance, along with significantly enhanced rewards. These empirical results demonstrate that our approach is not only theoretically sound but also practically effective, highlighting its potential as a transformative tool in the field of safe robot learning and beyond.

As for the fifth challenge (**Safety Benchmarks**), we develop benchmarks for both safe single-agent and multi-agent RL. Notably, we introduce three safe MARL benchmarks—Safe Multi-Agent MuJoCo (Safe MAMuJoCo), Safe Multi-Agent Robosuite (Safe MARobosuite), and Safe Multi-Agent Isaac Gym (Safe MAIG)—to enhance the toolkit available to the MARL and robotic control research communities.

The main contributions of this work are as follows: We present a comprehensive suite of safe RL methodologies, combining both theoretical insights and practical applications. First, we explore the trade-off between optimizing reward and safety in safe RL, proposing the PCRPO method based on gradient manipulation theory. Second, we tackle the challenge of safe multi-objective RL by introducing the CR-MOPO method, which ensures safety in robot learning while guaranteeing monotonic reward improvement. Third, we propose model-free MARL approaches for the safe control of multiple robots, validated through comprehensive robot control experiments. Fourth, to enable RL deployment in real-world scenarios, we develop the ROSCOM method, designed to mitigate safety violations under uncertain environmental disturbances using a manifold-based approach. Lastly, we contribute to the broader research community by creating a set of benchmarks for safe single-robot and multi-robot learning, facilitating fair and rigorous evaluations of safe robot learning performance. These contributions collectively advance the state of the art in safe RL and promote progress in research and practical applications.

This dissertation is organized as follows: The dissertation introduction, including an overview, motivation, challenges, and contributions are introduced in Chapter 1; foundations

of safe robot RL are presented in Chapter 2; the safe RL balance between reward and safety performance optimization is introduced in Chapter 3; the ensuring safety of multi-objective robot learning is introduced Chapter 4; safe multi-robot learning is provided in Chapter 5; safe robust robot RL is investigated in Chapter 6; safe robot RL benchmarks, including single-robot learning and multi-robot learning benchmarks, are introduced in Chapter 7; a conclusion for this dissertation is given in Chapter 8.

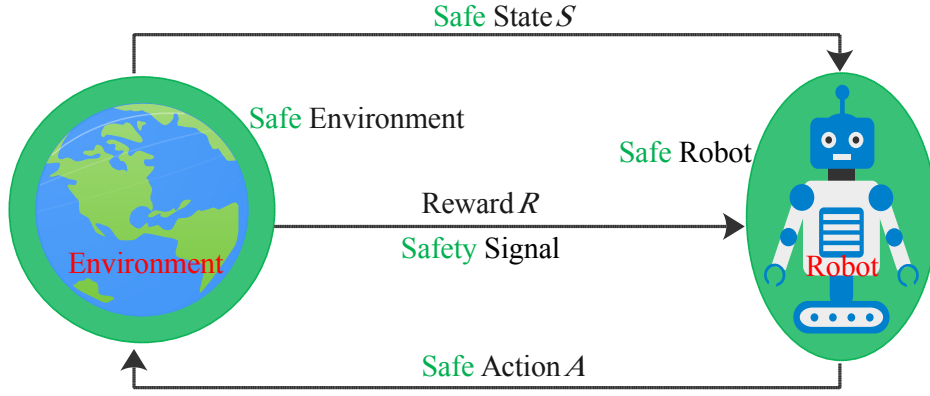


Figure 2.1: Safe robot reinforcement learning. Ensuring learning safety by modeling the agent’s safe action space, safe environment space, and safe state space, etc.

2

Foundations of Safe Robot Reinforcement Learning

In the domain of safe robot RL, as illustrated in Figure 2.1, the interaction between a robot (also referred to as an embodied agent) and its environment is a fundamental process. Within this framework, the robot performs safety-constrained actions that influence the environment, moving to safe states while receiving corresponding rewards and safety signals (costs) from the environment. This dynamic forms the core of the safe learning process, where the robot continuously adapts and refines its actions based on the feedback received from the environment to ensure both optimal reward performance and adherence to safety constraints.

Safe robot RL is often formulated as a Constrained Markov Decision Process (CMDP) [10], where the primary goal is to maximize the agent’s reward while adhering to predefined safety constraints. We begin by introducing the fundamentals of Markov Decision Processes (MDPs) and then delve into the specifics of CMDP settings. Following this, we provide a detailed formulation of safe RL problems, including a comprehensive discussion of objective functions relevant to safe RL frameworks. Finally, we introduce the existing safe RL methods. This chapter aims to highlight the key considerations essential for developing effective and reliable safe RL.

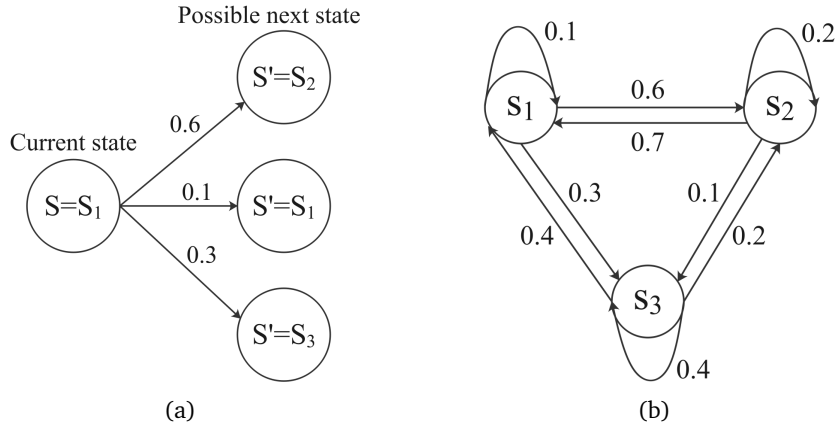


Figure 2.2: Markov transition processes. (a) From current states to next states. (b) a system's transition processes.

2.1 Markov Decision Processes

The Markov Decision Process (MDP) framework [234] has emerged as a well-established and widely adopted formulation in the field of RL in recent years. An infinite-horizon MDP $(\mathcal{S}, \mathcal{A}, P, r, \gamma)$ is specified by: a state space $\mathcal{S}$; an action space $\mathcal{A}$; a transition dynamics $P : \mathcal{S} \times \mathcal{A} \times \mathcal{S} \rightarrow [0, 1]$, where $P(s'|s, a)$ is the probability of transition from state $s \in \mathcal{S}$ to state $s' \in \mathcal{S}$ when action $a \in \mathcal{A}$ is taken; a reward function $r : \mathcal{S} \times \mathcal{A} \rightarrow \mathbb{R}$, where $r(s, a)$ is the instantaneous reward when taking action a in state s ; a discount factor $\gamma \in [0, 1)$.

2.1.1 Markov Processes

In the context of robotic systems, the historical sequence of states can be represented as $h(s_t) = \{s_1, s_2, \dots, s_t\}$, where t denotes the time step. The probability of transitioning to the next state s_{t+1} from the current state s_t is contingent upon the present state, formalized as $P(s_{t+1}|s_t) = P(s_{t+1}|h_t)$. This relationship is governed by the transition dynamics P , whereby the current state s evolves into another state s' . This process is mathematically articulated in Equation (2.1).

An illustrative example of this transition dynamic is depicted in Figure 2.2. In this example, various sample paths originating from state s_1 could include sequences like $s_1 \rightarrow s_2 \rightarrow s_1 \rightarrow s_3$, $s_1 \rightarrow s_1 \rightarrow s_2 \rightarrow s_2$, $s_1 \rightarrow s_3 \rightarrow s_3 \rightarrow s_2$, and so forth. These examples demonstrate the probabilistic nature of state transitions in robotic systems, highlighting the complexity and variability inherent in such processes.

$$P = \begin{pmatrix} P(s_1|s_1) & P(s_2|s_1) & \cdots & P(s_t|s_1) \\ P(s_1|s_2) & P(s_2|s_2) & \cdots & P(s_t|s_2) \\ \vdots & \vdots & \ddots & \vdots \\ P(s_1|s_t) & P(s_2|s_t) & \cdots & P(s_t|s_t) \end{pmatrix} \quad (2.1)$$

Building upon the aforementioned state transition dynamics, the reward value for a given state s_t can be estimated as $R(s_t = s) = \mathbb{E}[r_t | s_t = s]$. This allows for the formulation of a state value function $V_t(s)$, which represents the expected cumulative rewards from a specific time step t through to the terminal time step T .

$$V_t(s) = \mathbb{E}[R_{t+1} + \gamma R_{t+2} + \gamma^2 R_{t+3} + \dots + \gamma^{T-t-1} R_T | s_t = s]. \quad (2.2)$$

The value function V , commonly denoted as $V(s)$, can be effectively formulated using the Bellman equation. The Bellman equation provides a recursive decomposition of the value function, which is fundamental in RL. This recursive relationship is expressed as follows:

$$V_t(s) = \underbrace{R(s)}_{\text{Immediate reward}} + \underbrace{\gamma \sum_{s' \in S} P(s' | s) V(s')}_{\text{Discounted sum of future reward}} \quad (2.3)$$

$$V_t(s) = \mathbb{E}[R_{t+1} + \gamma \mathbb{E}[R_{t+2} + \gamma^2 R_{t+3} + \dots] | s_t = s] \quad (2.4)$$

The state value function $V_t(s)$ in RL can indeed be expressed in a matrix form, as shown in Equation (2.5). This representation is particularly useful for illustrating the relationship between various states and their corresponding values. In matrix notation, the value function is typically laid out as a vector, where each element corresponds to the expected cumulative reward for being in a particular state. This matrix form provides a structured and efficient way of representing the value function across different states within the state space.

$$\begin{bmatrix} V(s_1) \\ V(s_2) \\ \vdots \\ V(s_N) \end{bmatrix} = \begin{bmatrix} R(s_1) \\ R(s_2) \\ \vdots \\ R(s_N) \end{bmatrix} + \gamma \begin{bmatrix} P(s_1|s_1) & P(s_2|s_1) & \cdots & P(s_N|s_1) \\ P(s_1|s_2) & P(s_2|s_2) & \cdots & P(s_N|s_2) \\ \vdots & \vdots & \ddots & \vdots \\ P(s_1|s_N) & P(s_2|s_N) & \cdots & P(s_N|s_N) \end{bmatrix} \begin{bmatrix} V(s_1) \\ V(s_2) \\ \vdots \\ V(s_N) \end{bmatrix} \quad (2.5)$$

2.1.2 Markov Reward Processes with Decisions

Derived from the framework of Markov processes, the concept of robot action is introduced. Specifically, once a robot receives information from its environment, it selects an action a from a set of possible actions A . This action is then executed within the environment with the objective of maximizing the robot's cumulative reward. This decision-making process is a fundamental aspect of robot RL, where the robot continuously interacts with its environment, making sequential decisions to optimize its performance based on the feedback received.

Incorporating the decision policy π into the transition dynamics, with initial transition distribution ρ_0 , the process can be represented by the equation detailed in Equation (2.6) and Figure (2.3). This equation integrates the decision policy π , which defines the probability of choosing a particular action a in a given state s , into the Markov decision process. The policy π thus plays a crucial role in influencing the state transitions based on the actions taken, guiding the robot's behavior in pursuit of its objective to maximize cumulative rewards.

$$P^\pi(s' | s) = \sum_{a \in A} \pi(a | s) P(s' | s, a) \quad (2.6)$$

The reward function and the state value function, when factoring in the decision policy π , can be reformulated as follows:

$$\begin{aligned} R^\pi(s) &= \sum_{a \in A} \pi(a | s) R(s, a) \\ V^\pi(s) &= \mathbb{E}_\pi[R_{t+1} + \gamma R_{t+2} + \gamma^2 R_{t+3} + \dots | s_t = s] = \mathbb{E}_\pi \left[\sum_{t=0}^{\infty} \gamma^t r(s_t) | s_t = s \right] \end{aligned} \quad (2.7)$$

The reward function under policy π is defined to reflect the expected reward for taking action a in state s according to policy π . Mathematically, it can be expressed as $R^\pi(s, a)$, representing the expected reward when the policy π is adhered to. Similarly, the state value function under policy π , denoted as $V^\pi(s)$, is recalculated to represent the expected cumulative

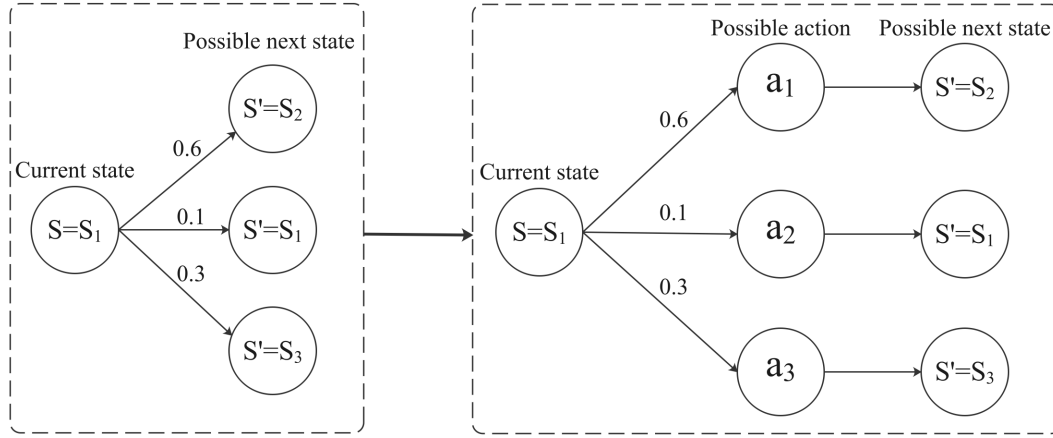


Figure 2.3: Markov reward processes with decisions.

reward from state s while following policy π . This function takes into account the probabilistic nature of the policy's action selection and the corresponding transitions and rewards. These reformulations provide a more nuanced understanding of the reward dynamics and the value of states under a specific policy, thus enabling a more targeted approach to optimizing the decision-making process in RL scenarios.

Expanding upon the concept of the state-value function $V^\pi(s)$, we introduce the action-value function $Q^\pi(s, a)$, as shown in Equation (2.8). The action-value function, $Q^\pi(s, a)$, is defined as the expected return (cumulative reward) when the agent starts in state s , takes an action a , and subsequently follows policy π for all future decisions. This function provides a measure of the expected effectiveness of each action in each state, under the guidance of a given policy π . It serves as a fundamental component in the optimization of decision-making processes within the RL framework.

$$Q^\pi(s, a) = \mathbb{E}_\pi \left[\sum_{t=0}^{\infty} \gamma^t r(s_t, a_t) \mid s_t = s, a_t = a \right] \quad (2.8)$$

The relationship between the state-value function V and the action-value function Q is a important concept in RL. This relationship is mathematically articulated in Equation (2.9). Typically, this relationship is expressed through a formula that defines the state-value function $V^\pi(s)$ in terms of the action-value function $Q^\pi(s, a)$, under a given policy π . This equation encapsulates how the value of being in a particular state under a policy is related to the values of the actions that can be taken in that state, also under the same policy.

$$V^\pi(s) = \sum_{a \in A} \pi(a \mid s) Q^\pi(s, a) \quad (2.9)$$

Based on the established relationship between the state value function $V^\pi(s)$ and the action value function $Q^\pi(s, a)$, it is possible to decompose these functions to gain further insights into their dynamics. This decomposition process involves breaking down the state value function into its constituent parts based on the action value function.

Typically, the state value function $V^\pi(s)$ is expressed as the expected value of the action value function $Q^\pi(s, a)$ under policy π . This reflects the expected return of being in state s and taking an action according to policy π , which is averaged over all possible actions. Such a decomposition is crucial for understanding how the value of a state under a certain policy is influenced by the potential actions available in that state, thereby providing a foundational framework for RL algorithms to optimize policy.

$$\begin{aligned}
V^\pi(s) &= E_\pi [R_{t+1} + \gamma V^\pi(s_{t+1}) \mid s_t = s] \\
Q^\pi(s, a) &= E_\pi [R_{t+1} + \gamma Q^\pi(s_{t+1}, a_{t+1}) \mid s_t = s, a_t = a] \\
Q^\pi(s, a) &= R(s_t, a_t) + \gamma \sum_{s_{t+1} \in S} P(s_{t+1} \mid s_t, a_t) V^\pi(s_{t+1})
\end{aligned} \tag{2.10}$$

In the process of policy improvement within RL, the utilization of a greedy strategy, $\pi'(s) = \text{greedy}(V^\pi)$, is a common approach for maximizing reward performance, that is $\pi'(s) = \underset{a}{\operatorname{argmax}} Q^\pi(s, a)$ with a probability. This strategy involves selecting actions that promise the highest immediate rewards based on the current understanding of the environment, focusing on optimizing short-term gains.

Concurrently, this approach incorporates an element of exploration to gather unknown information, which is essential for uncovering potentially more advantageous actions that have not yet been explored. The key objective in employing a greedy strategy in policy improvement is to enhance the overall effectiveness of the policy π by maximizing the state value function $V^\pi(s)$. This maximization process leads to the refinement of the policy π , as it iteratively adjusts to select actions that increase the expected cumulative rewards in each state. Balancing the aspects of exploitation (using known information to maximize rewards) and exploration (acquiring new information) is crucial in this context, ensuring that the policy does not solely rely on immediate rewards but also adapts to the broader dynamics of the environment for long-term optimization.

Leveraging the greedy strategy, policy evaluation and improvement can be systematically achieved through iterative updates [234]. The core concept of this approach is to incrementally refine the policy by selecting actions that maximize the expected rewards, as per the current understanding of the state-value or action-value functions with policy evaluation and improvement. These updates are typically represented by a set of equations that capture the essence of the iterative improvement process. Thus, we can observe,

$$Q^\pi(s, \pi'(s)) = \max_{a \in A} Q^\pi(s, a) \geq Q^\pi(s, \pi(s)) = V^\pi(s)$$

$$\begin{aligned}
V^\pi(s) &\leq Q^\pi(s, \pi'(s)) = \mathbb{E}_{\pi'} [R_{t+1} + \gamma V^\pi(s_{t+1}) \mid S_t = s] \\
&\leq \mathbb{E}_{\pi'} [R_{t+1} + \gamma Q^\pi(s_{t+1}, \pi'(s_{t+1})) \mid S_t = s] \\
&\leq \mathbb{E}_{\pi'} [R_{t+1} + \gamma R_{t+2} + \gamma^2 Q^\pi(s_{t+2}, \pi'(s_{t+2})) \mid S_t = s] \\
&\leq \mathbb{E}_{\pi'} [R_{t+1} + \gamma R_{t+2} + \dots \mid S_t = s] = V_{\pi'}(s)
\end{aligned}$$

Therefore, we have the Bellman optimality equations, the Bellman optimality equations form the cornerstone of finding optimal policies in RL. These equations provide a recursive decomposition for the optimal state-value and action-value functions. The Bellman optimality equations can be outlined as follows:

Optimal State-Value Function: This equation describes the maximum expected return achievable from any given state under an optimal policy. It is typically expressed as:

$$V^*(s) = \max_a \sum_{s' \in S} P(s' \mid s, a) [r + \gamma V^*(s')]$$

Here, $V^*(s)$ represents the optimal state-value function, γ is the discount factor, and the sum is taken over all possible next states s' and rewards r , with $P(s' \mid s, a)$ denoting the transition probability.

Optimal Action-Value Function: This equation extends the concept to the action-value function, providing the maximum expected return for taking an action a in state s under the

optimal policy:

$$Q^*(s, a) = \sum_{s' \in \mathcal{S}} P(s'|s, a) [r + \gamma \max_{a'} Q^*(s', a')]$$

In this equation, $Q^*(s, a)$ is the optimal action-value function, and the maximization is over all possible actions a' in the subsequent state s' .

These equations are important to many algorithms in RL, serving as a theoretical foundation for finding optimal policies through various methods of dynamic programming, Monte Carlo methods, and temporal-difference learning.

2.2 Constrained Markov Decision Processes

A Markov Decision Process (CMDP) problem [10] is an extension of a standard MDP with a constraint set $\mathcal{C}$. A tuple $(\mathcal{S}, \mathcal{A}, P, r, \rho_0, \gamma)$ is given to present a MDP [204]. A state set is denoted as $\mathcal{S}$, an action set is denoted as $\mathcal{A}$, $P(s'|s, a)$ denotes the probability of state transition from s to s' after playing a . A reward function is denoted as $r : \mathcal{S} \times \mathcal{A} \times \mathcal{S} \rightarrow \mathbb{R}$. $\rho_0(\cdot) : \mathcal{S} \rightarrow [0, 1]$ is the starting state distribution, γ denotes the discount factor.

In safe RL, the goal of an optimal policy π is to maximize the reward and minimize the cost by selecting an action a , and $\Pi_{\mathcal{S}}$ is the policy set, $\mathcal{T}$ is a trajectory, $\mathcal{T} = (s_0, a_0, s_1, \dots)$, in which an action depends on π , $s_0 \sim \rho_0(\cdot)$, $a_t \sim \pi(\cdot|s_t)$, $s_{t+1} \sim P(\cdot|s_t, a_t)$. $d_{\pi}^{s_0}(s) = (1-\gamma) \sum_{t=0}^{\infty} \gamma^t P_{\pi}(s_t = s|s_0)$ denotes the state distribution (starting at s_0), the discounted state distribution based on the initial distribution $\rho_0(\cdot)$ is present as $d_{\pi}^{\rho_0}(s) = \mathbb{E}_{s_0 \sim \rho_0(\cdot)}[d_{\pi}^{s_0}(s)]$.

The state value function is shown in Function (2.11), the state action value function is shown in Function (2.12); the advantage function is shown in Function (2.13); the reward objective is shown in Function (2.14).

$$V_{\pi}(s) = \mathbb{E}_{\pi} \left[\sum_{t=0}^{\infty} \gamma^t r_{t+1} | s_0 = s \right]. \quad (2.11)$$

$$Q_{\pi}(s, a) = \mathbb{E}_{\pi} \left[\sum_{t=0}^{\infty} \gamma^t r_{t+1} | s_0 = s, a_0 = a \right]. \quad (2.12)$$

$$A_{\pi}(s, a) = Q_{\pi}(s, a) - V_{\pi}(s). \quad (2.13)$$

$$J(\pi) = \mathbb{E}_{s \sim \rho_0(\cdot)} [V_{\pi}(s)]. \quad (2.14)$$

CMDP is based on MDP, in which the additional constraint set $\mathcal{C} = \{(C_j, c_j)\}_{j=1}^m$ is considered, where C_j is the cost value functions, and c_j is the safety constraint bound, $j \in [m]$, m is the type number of cost constraints. Similarly, we have the cost value functions $V_{C_j}^{\pi}$, cost action-value functions $Q_{C_j}^{\pi}$, and cost advantage functions $A_{C_j}^{\pi}$, e.g., $V_{C_j}^{\pi}(s) = \mathbb{E}_{\pi} \left[\sum_{t=0}^{\infty} \gamma^t C_j(s_t, a_t) | s_0 = s \right]$. Based on the above definition, we have the expected cost function $C_j(\pi) = \mathbb{E}_{s \sim \rho_0(\cdot)} [V_{C_j}^{\pi}(s)]$. The feasible policy set $\Pi_{\mathcal{C}}$ that can satisfy the safety constraint bound is as follows,

$$\Pi_{\mathcal{C}} = \cap_{j=1}^m \{ \pi \in \Pi_{\mathcal{S}} \text{ and } C_j(\pi) \leq c_j \}.$$

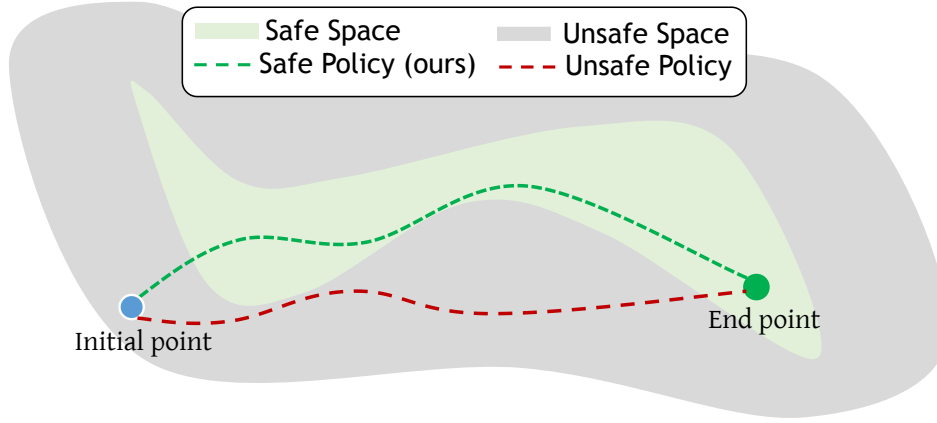


Figure 2.4: Safe policy for robot learning (This figure is derived from [93]).

The goal of safe RL is to maximize the reward performance and minimize the cost values to guarantee safety. As shown in Figure 2.4, the agent should explore the environments within a safe space to search for a safe policy.

$$\max_{\pi \in \Pi_S} J(\pi), \text{ such that } \mathbf{C}(\pi) \preceq \mathbf{c}, \quad (2.15)$$

where the vector $\mathbf{C}(\pi) = (C_1(\pi), C_2(\pi), \dots, C_m(\pi))^T$, and $\mathbf{c} = (c_1, c_2, \dots, c_m)^T$. Therefore, the solution of a CMDP problem (2.15) can be provided as the optimization function (2.16):

$$\pi_* = \arg \max_{\pi \in \Pi_C} J(\pi). \quad (2.16)$$

2.3 Existing Safe Reinforcement Learning Methods

In this section, we provide a concise but holistic overview of safe RL methods from a bird's eye view and attempt to discuss the existing solutions for the CMDP problem in safe RL. In particular, we will introduce safe RL methods from the perspectives of model-based methods and model-free methods, in which safe single-agent RL and multi-agent RL will be analyzed in detail. In particular, we will introduce policy optimization-based approaches, formal methods-based approaches, control theory-based approaches, and Gaussian processes-based approaches in model-based and model-free settings, respectively.

2.3.1 Methods of Model-Based Safe Reinforcement Learning

Although accurate models are challenging to build and many applications lack models, model-based Deep RL (DRL) methods usually have a better learning efficiency than model-free DRL methods. There are still many scenarios for which we can apply model-based DRL methods, such as robotics, transportation planning, logistics, etc. Several works have shown that safe problems, such as a safe robot control problem [241], can be overcome by using model-based safe RL methods. In this section, we will investigate model-based safe RL methods from different perspectives: policy optimization-based approaches, control theory-based approaches, formal methods-based approaches, and Gaussian process-based approaches.

2.3.1.1 Policy optimization-based approaches

Policy optimization-based approaches usually search for a safe policy using cumulative cost values on trajectories. For example, Moldovan and Abbeel use the Chernoff function [179] to achieve near-optimal bounds with desirable theoretical properties. Different from the Chernoff function based methods [179], Borkar [33] proposes an actor-critic RL method to handle a CMDP problem based on the envelope theorem in mathematical economics [169, 173], in which the primal-dual optimization is analyzed in detail using a three-time scale process. The critic scheme, actor scheme, and dual ascent are on the fast, middle, and slow timescale. Bharadhwaj *et al.* [29] also present an actor-critic method to address a safe RL problem, where they first develop a conservative safety critic to estimate the safety. The primal-dual gradient descent is leveraged to optimize the reward and cost value by constraining the failure probability. Although this method can bound the probability of failures during policy evaluation and improvement, this method still cannot guarantee total safety; a few unsafe corner actions may dramatically damage critical robot applications.

Akin to Borkar’s method [33], Tessler *et al.* [242] also utilize a multi-timescale approach with regards to cost as a part of the reward in primal-dual methods. However, the method’s learning rate is hard to tune for real-world applications because of imposing stringent requirements, and the method may not guarantee safety when agents are training.

Similar to the above methods, in actor-critic settings, with primal-dual optimization techniques, Yu *et al.* [278] convert a non-convex constrained problem into a locally convex problem and guarantee the stationary point to the optimal point of non-convex optimization problem; they consider the state-action safety optimization, and the optimization process is motivated by [146], in which the Lipschitz condition is necessary to satisfy the results. Also, they need to estimate the policy gradient for optimization.

Analogously, using optimization theory, the policy gradient and actor-critic methods are proposed by Chow *et al.* [50] to optimize risk RL performance, in which CVaR¹-constrained and chance-constrained optimization are used to guarantee safety. Specifically, the importance sampling [18, 237] is leveraged to improve policy estimation, and convergence analysis for the proposed algorithms is provided. Nevertheless, this method may not guarantee safety during training [290]. Paternain *et al.* [194] provide a duality theory for CMDP optimization, and they prove the zero duality gap in primal-dual optimization even for non-convexity problems. Furthermore, they point out that the primal problem can be exactly solved by dual optimization.

2.3.1.2 Control theory-based approaches

Control theory-based approaches mostly ensure more rigorous safety than policy optimization approaches. Nevertheless, control theory-based approaches may not be easy to transfer to other domains due to the strict requirements of dynamics models. For instance, Berkenkamp *et al.* [26] develop a safe model-based RL algorithm by leveraging Lyapunov functions to guarantee stability with the assumptions of Gaussian process prior; their method can ensure a high-probability safe policy for an agent in a continuous process. However, Lyapunov functions are usually hand-crafted, and it is not easy to find a principle to construct Lyapunov functions for an agent’s safety and performance [51]. In addition, some safe RL methods are proposed from the perspective of Model Predictive Control (MPC) [281], e.g., MPC is used to make robust decisions in CMDPs [15] by leveraging a constrained MPC method [172], which also introduces a general safety framework to make decisions [77].

Apart from Lyapunov functions and MPC-safe RL approaches, control barrier functions (CBFs) based approaches are also proposed to ensure learning safety [49, 58, 74, 162]. Similar

¹CVaR denotes the Conditional Value at Risk.

to the above control methods, CBFs based safe RL approaches also require a dynamics model for the control system. Thus, it is not straightforward to deploy it in RL [168]. For example, [162] propose a model-based safe RL by leveraging lagrangian optimization and CBFs; they further deploy their method in simulation and real-world autonomous driving experiments, the experiment results indicate that their method can improve sample efficiency and ensure safety. In contrast to [162], [74] not just use CBFs, but also robust CBFs [73], they develop a safe RL method based on SAC [96] and robust CBFs. Specifically, they consider safety in robust settings by estimating the disturbed dynamics systems with Gaussian Proces (GP) models. [58] propose a safe exploration framework based on model-based RL and CBFs, where Lyapunov-like CBFs [190] are used to construct the safety CBFs, and they can ensure the learning safety and stability. The above methods show impressive performance in ensuring safety. However, designing CBFs necessitates knowledge of the model or safety certificates, which may be challenging in complex environments, even when the model is derived using alternative methods.

2.3.1.3 Formal methods-based approaches

Different from policy optimization-based approaches and control theory-based approaches, formal methods [13] usually try to ensure safety without unsafe probabilities. However, most formal methods rely heavily on the model knowledge and may not show better reward performance than other methods. The verification computation might be expensive for each neural network [13]. More generally, the curse of dimensionality problem is challenging to be solved, which appears when formal methods are deployed for RL safety [22], since formal methods may be complex to verify RL safety in continuous and high-dimension space settings [22].

For instance, in [13], Anderson *et al.* provide a neurosymbolic RL method by leveraging formal verification to guarantee RL safety in a mirror descent framework. The experiment results show promising performance in RL safety, though the fixed worst-case model knowledge is used in these environments, which may not be suitable for practical applications. Similarly, in [22], Beard and Baheri utilize formal methods to improve agent safety by incorporating external knowledge and penalizing behaviors for RL exploration. Nonetheless, the methods may need to be developed for scalability and continuous systems. Finally, in [80], Fulton and Platzer also use external knowledge to ensure agent safety by leveraging the justified speculative control sandbox in offline formal verification settings.

2.3.1.4 Gaussian processes-based approaches

In formal methods, determining how to measure unsafe areas is a challenging problem. Recently, many approaches have been proposed by leveraging a GP [258] to estimate the uncertainty and unsafe areas. Further, the information from GP methods is incorporated into the learning process for agent safety. For instance, Akametalu *et al.* [7] develop a safe RL method based on reachability analysis, in which they use GP methods to measure the disturbances which may lead to unsafe states for agents, the maximal safe areas are computed iteratively in an unknown dynamics system. Like Akametalu *et al.* [7] method, Berkenkamp and Schoellig [25] utilize GP methods to measure the system uncertainty and further guarantee system stability. Polymenakos *et al.* [203] develop a safe policy search approach based on PILCO (Probabilistic Inference for Learning Control) method [63] which is a policy gradient method derived from a GP, in which they improve agent safety using probability trajectory predictions by incorporating cost into reward functions. Similar to Polymenakos *et al.* [203], Cowen *et al.* [59] also use PILCO to actively explore environments

while considering risk, a GP is used to quantify the uncertainty during exploration, and agent safety probability is improved by leveraging a policy multi-gradient solver.

The above safe RL methods present excellent performance in terms of the balance between reward and safety performance in most challenging tasks. Nonetheless, training safety or stability may need to be further investigated rigorously, and a unified framework may need to be proposed to better examine safe RL performance.

2.3.2 Methods of Model-Free Safe Reinforcement Learning

Most studies pay attention to model-free safe RL since it can be deployed in many domains without requiring model dynamics. In this section, we also investigate safe RL methods from the perspectives of policy optimization, control theory, Gaussian processes, and formal methods.

2.3.2.1 Policy optimization-based approaches

Constrained Policy Optimisation (CPO) [4] is a classical policy gradient method to solve the CMDP problem based on model-free deep RL. In particular, a policy has to be optimized to guarantee the reward of a monotonic improvement while satisfying safety constraints. As a result, their methods can almost converge to safety bound and provide more comparable performance than the primal-dual method [50] on some tasks. However, CPO's computation is more expensive than PPO²-Lagrangian, since it needs to compute the Fisher information matrix and uses the second Taylor expansion to optimize objectives. Moreover, the approximation and sampling errors may have detrimental effects on the overall performance, and the convergence analysis is challenging.

Derived from CPO [4], Projection-based Constrained Policy Optimisation (PCPO) [271] based on two-step methods constructs a cost projection to optimize cost and guarantee safety, which displays better performance than CPO on some tasks. PCPO leverages policy to maximize the reward via Trust Region Policy Optimization (TRPO) method [216], and then projects the policy to a feasible region to satisfy safety constraints. However, the second-order proximal optimization is used in both steps, which may result in a more expensive computation than the First Order Constrained Optimization in Policy Space (FOCOPS) method [290], which only uses the first-order optimization. [290] is motivated by the optimization-based idea [199], where they use the primal-dual method, address policy search in the nonparametric space and project the policy into the parameter space, to carry out proximal maximization optimization in CMDPs. Although this method is easy to implement and shows better sample efficiency, it still needs to solve the problems of unstable saddle points and unsafe actions during training.

Similar to CPO [4], in Safe Advantage-based Intervention for Learning policies with Reinforcement (SAILR) [255], Wagener *et al.* leverage the advantage function as a surrogate to minimize cost, and further achieve safe policy both during training and deployment. Furthermore, In [229], a Shortest-Path Reinforcement Learning (SPRL) method is proposed using off-policy strategies to construct safe policy and reward policy, and its applications are used for the shortest path problems in Travel Sale Path (TSP). Besides, based on a Gaussian process of a safe RL method [254], the SNO-MDP³ [253] is developed to optimize cost in the safe region and optimize the reward in the unknown safety-constrained region [231, 247]. This method [253] shows the near-optimal cumulative reward under some assumptions, whereas it cannot achieve the near-optimal values while guaranteeing safety constraints.

²PPO denotes the Policy Proximal Optimization.

³The SNO-MDP represents the Safe Near-Optimal MDP.

Different from CPO, a first-order policy optimization method [152] is provided based on interior point optimization (IPO) [37], in which the logarithmic barrier functions are leveraged to satisfy safety constraints. The method is easy to implement by tuning the penalty function. Although the empirical results on MuJoCo [4] and grid-world environments [53] have demonstrated their method's effectiveness, the theoretical analysis to guarantee the performance is still needed to be provided.

Unlike the above CMDP optimization, a meta algorithm [177] is proposed to solve safe RL problems with general convex constraints [38]. They can, in particular, solve CMDP problems with a small number of samples in reward-free settings. The algorithms can also be used to solve approachability problems [30, 176]. Although the theoretical results have shown their method's effectiveness, practical algorithms and experiments ought to be proposed and carried out to further evaluate their method effectiveness. In [193], an attempt is made to solve safe RL using the trajectory probability in a safe set. The probability invariance is positive, and constraint gradients are obtained using the policy parameters. More importantly, the related problem can have an arbitrarily small duality gap. However, this method may also encounter the stability problems of primal-dual methods, and the saddle points using Lagrangian methods may be unstable [189].

Comparable to the above primal-dual settings, Ghosh *et al.* [84] have developed a safe RL method based on least-squares value iteration-upper confidence bound [114] in primal-dual settings, which is suitable for large-scale optimization, and the regret bound and safety violation are analyzed. While similar methodologies have been proposed, they are predominantly utilized within tabular settings, as exemplified by the work of Wei *et al.* [257]. Furthermore, Xiong *et al.* [264] introduce a step-wise safe RL method based on upper confidence bound value iteration [16]. This method employs optimistic estimations of transition kernels and cost functions, thereby enhancing its performance to ensure safety in reward-free scenarios. Notably, Xiong *et al.* also provide analyses of safety violations and regret bounds, facilitating the extension of this method to additional contexts such as multi-robot systems. Although these methods demonstrate improved performance over related baselines, further exploration into their applications across diverse domains, such as safe multi-robot control and planning, could yield even more beneficial insights. Moreover, in recent years, primal-based methods [266] have emerged as powerful alternatives to primal-dual-based approaches [4, 268], demonstrating promising performance in practical applications. Notable examples include CRPO [266] and PCRPO [92]. A significant advantage of primal-based methods is that they do not require tuning dual parameters and are not influenced by initialization [266]. Specifically, CRPO introduces a primal-based framework for safe RL, but it may have experienced oscillations between reward and safety optimization, potentially undermining its performance. To address this issue, PCRPO provides a soft policy optimization technique with gradient manipulation, exhibiting superior performance compared to CRPO and SOTA primal-dual-based methods such as PCPO [271], CUP [268], and PPOLag [207].

2.3.2.2 Control theory-based approaches

In contrast to safe policy optimization such as CPO, PCRPO, and IPO, control theory is also leveraged to ensure RL safety. For example, the first Lyapunov functions used for safe RL may be [198], in which the agent's actions are constrained by applying the control law of Lyapunov functions to learning systems and removing the unsafe actions in the action set. The experiments have demonstrated that their method can achieve safe actions for the control problems in their study. However, the method requires the knowledge of a Lyapunov function in advance. If the environment dynamics model is unknown, it may be difficult to address safe RL problems with this method. Unlike [198], in [51] and [52], Chow *et al.* propose several safe RL methods based on Lyapunov functions in discrete and continuous CMDP settings,

respectively, where Lyapunov functions are used for safe RL to improve learning safety and stability. The methods can guarantee safety during training, and the Lyapunov functions can be designed by a proposed Linear Programming algorithm. However, the training stability and safety using Lyapunov functions still need to be improved, and more efficient algorithms in the setting may need to be proposed.

Apart from Lyapunov functions based safe RL, CBFs are also developed in model-free settings in recent years [252, 273], [273] introduces a model-free safe RL method based CBFs, in which they learn the policy and CBFs with data-driven methods, which can help to reduce reliance on the environment models. Similarly, [252] proposes a safe RL method for power system control with CBFs, in which they integrated CBFs into reward functions to search for safe policy. model free control barrier function, [167], [283]. Moreover, [46] leverages a model-free RL and CBFs to guarantee learning safety while improving learning efficiency, in which CBFs are used to constrain the search space to enhance learning safety and efficiency based on a GP learning model. Furthermore, [168] also leverages CBFs to ensure learning safety with learned dynamics knowledge, where they provide convergence and stability analysis, and their method outperforms the baselines in their experiments.

2.3.2.3 Gaussian processes-based approaches

Relative to safe policy optimization methods and Lyapunov function-based techniques, GP methodologies primarily concentrate on the safe exploration via modeling the safe states. In a GP of model-free settings, Sui *et al.* [231] use a GP method to present the unknown reward function from noise samples, the exploration by leveraging a GP method is improved to reduce uncertainty and ensure agent safety. More particularly, the GP method is used to predict unknown function, and guide the exploration in bandit settings which do not need to state transitions. Their real-world experiments in movie recommendation systems and medical areas indicate that their method can achieve near-optimal values safely. Like Sui *et al.* [231], Turchetta *et al.* [247] also leverage a GP method to approximate unknown functions prior for safe exploration. Nevertheless, they focus more on finite MDP settings considering explicitly reachability. Nonetheless, the method may not optimize reward objectives while considering safety. Wachi *et al.* [254] represent unknown reward and cost functions with GP methods to ensure safety with probability and optimize reward. Furthermore, the safe, unsafe, and uncertain states are denoted for agent optimistic and pessimistic exploration, and their method can adapt the trade-off between exploration and exploitation. However, the convergence guarantee with finite-time rates, optimization for multiple and heterogeneous objectives may need to be provided. Although GP methods have shown impressive performance with regard to RL safety, most of them ensure safety with probability.

2.3.2.4 Formal methods-based approaches

Distinct from employing GP methods to model the safe state, Hasanbeig *et al.* [99] propose a safety-enhanced RL approach that leverages LTL. In this methodology, logical formulas serve as constraints during the exploration phase of policy synthesis. Although this method demonstrates notable safety performance, it is imperative to carefully define the logical constraints to ensure safe exploration and effectively balance the trade-off between safety performance and reward acquisition. Murugesan *et al.* [181] employ a formal tool, satisfiability modulo theories [56], as a safety verification layer to enhance learning safety. Furthermore, Hasanbeig *et al.* [245] encode properties of LTL into the reward function through reward engineering using a limit deterministic Büchi automaton [223] to ensure safety. While the aforementioned methods exhibit impressive performance in ensuring safety in most tasks, their deployment in real-world applications still needs to be further investigated.

2.3.3 Safe Multi-Agent Reinforcement Learning

Most current RL methods are based on the single-agent setting. Safe MARL is still a relatively new area that has emerged in recent years. Little research has yet been carried out that considers the safe multi-agent RL, which can be seen as a multi-agent CMDP problem. Safe multi-agent RL not only needs to consider the ego agent's safety and other agents' safety, but also needs to take into account the ego agent's reward and other agents' reward. In this section, we briefly introduce some popular safe multi-agent RL methods.

The Multi-Agent Constrained Policy Optimisation (MACPO) algorithm [90] is the first safe model-free MARL algorithm which is developed based on CPO [4] and Heterogeneous-Agent Trust Region Policy Optimisation (HATRPO) [131]; it can guarantee monotonic improvement in reward, while theoretically satisfying safety constraints during each iteration. The algorithm was tested on several challenging tasks, e.g. safe multi-agent Ant tasks, safe multi-agent Lift tasks. However, MACPO is an expensive computation algorithm since the Fisher Information Matrix is computed to approximate conjugate gradients. In contrast to MACPO, MAPPO-Lagrangian [90] is the model-free safe MARL algorithm that requires less time for computation on most of challenging tasks, and is easily implemented. [87] presents a soft-constrained policy optimization approach for real-world robot control, which investigates sim-to-real issues regarding deploying safe RL in real-world applications. Despite these advancements, further investigation is required to ensure RL robust safety.

Safe Decentralized Policy Gradient (Dec-PG) [158] using a decentralized policy descent-ascent method is the most closely-related algorithm to MACPO. The saddle point considering reward and cost is searched by a primal-dual framework in a multi-agent system. However, a consensus network used in this method imposes an extra constraint of parameter sharing among neighboring agents, which could yield suboptimal solutions [131]. In contrast, MACPO does not require the assumption of parameter sharing. Furthermore, multi-agent policy gradient methods can suffer from high variance by parameter sharing [132].

Another recently safe MARL work is CMIX [148], which considers peak and average constraints using QMIX [206]. Even though CMIX can satisfy the multi-agent cost constraint in these experiments, it does not provide theoretical analysis to guarantee safety for a multi-agent system. CMIX is also the parameter-sharing algorithm that suffers from problems similar to those of Safe Dec-PG [158].

Moreover, based on formal methods [260], Safe MARL via shielding is proposed [72], in which they study the safe MARL problem by using one of the formal methods, Linear Temporal Logic (LTL) [201] that is already adopted in safe single-agent settings [8]. Specifically, an attempt is made to ensure the safety of multi-agent systems by leveraging central shielding or factored shielding methods, which are used to specify the safe actions that agents only can take, and also correct the unsafe actions that agents explored. Furthermore, they evaluate the methods based on Multi-Agent Deep Deterministic Policy Gradient (MADDPG) [157] and Coordinating Q-learning (CQ-learning) methods [62]. Although this method can guarantee safety for multi-agent systems to some extent, it requires a prior LTL safety specification and problem-specific knowledge in advance. In a similar way that [72] ensures safety by optimizing the exploration, [222] introduces a safe MARL algorithm by adding a safety layer based on safe DDPG [60] and MADDPG [157] for continuous action space settings. Their method significantly improved safety in certain MARL tasks; however, ensuring zero violations across all tasks may be challenging.

2.3.4 Summary

In this section, we introduce safe RL algorithms from different perspectives, such as safe single-agent RL, safe multi-agent RL; model-based safe RL, model-free safe RL. Even though a number of algorithms display impressive performance in terms of reward scores, there is a long way to go toward real-world applications, and safe MARL is still a relatively new area that requires a lot more attention.

Safe Balanced Robot Reinforcement Learning

Ensuring RL safety is crucial for its deployment in real-world applications. Nevertheless, managing the trade-off between reward and safety during exploration presents a significant challenge. Improving reward performance through policy adjustments may adversely affect safety performance. In this study, we aim to address this conflicting relation by leveraging gradient manipulation. Initially, we analyze the conflict between reward and safety gradients. Subsequently, we tackle the balance between reward and safety optimization by proposing a soft switching policy optimization method, for which we provide convergence analysis. Based on our theoretical examination, we provide a safe RL framework to overcome the aforementioned challenge. Finally, we evaluate the effectiveness of our method on the Safety-MuJoCo Benchmark and a popular safe RL benchmark, Omnisafe. Experimental results demonstrate that our algorithms outperform several SOTA baselines in terms of balancing reward and safety optimization.

3.1 Introduction of Safe Balanced Robot Reinforcement Learning

Balancing the optimization of rewards and safety is a crucial aspect of enhancing robot learning performance. This balance addresses the inherent conflict that often arises during safe robot learning, where the pursuit of maximizing rewards may sometimes conflict with the imperative of maintaining safety. Achieving an optimal balance between these two objectives is key to developing effective and reliable robot learning systems.

Several methods are proposed to handle the balance between reward and safety optimization. For instance, the works of Zhou et al. [294] and Calian et al. [42] present impressive performance for ensuring learning safety, though there are some limitations. These include the absence of guarantees regarding monotonic performance improvement, as well as the presence of gradient conflicts between reward and cost, potentially impacting the effectiveness of their methods. Additionally, their approaches fall within the category of primal-dual optimization, where the initial policy’s quality could negatively impact performance. In contrast, our approach is designed to address these shortcomings. Furthermore, when comparing our method to Calian et al.’s [42], it notes that while their method focuses on handling soft constraints, ours is specifically tailored to address hard constraints, providing a rigorous assurance of safety.

Moreover, Constrained Policy Optimization (CPO) [5] is developed to ensure reward monotonic improvement while maintaining safety. PPO Lagrangian and TRPO Lagrangian

methods [208] are introduced to address the balance between reward and safety performance by employing Lagrangian optimization. Additionally, safe exploration methods based on a Gaussian Process (GP) [231] are introduced to guarantee exploration safety by utilizing a GP to model the exploration safety of the state space. However, these methods may not effectively resolve the conflict reward and cost gradients, and balance reward and safety optimization. A key question that is raised in this domain is: How can we handle the balance between reward and safety optimization?

In this research, we aim to address the key question by leveraging the theory of gradient manipulation, wherein we conduct a detailed examination of the changes in gradients associated with reward and safety. Based on our theoretical analysis, we propose the projection constraint-rectified policy optimization (PCRPO) method, designed to alleviate the conflict between reward and safety optimization while maintaining a balance between their optimization levels, in which soft switching policy optimization through gradient manipulation is proposed and a slack technique is introduced for adjusting the emphasis on safety optimization. Particularly, we evaluate the effectiveness of our method on challenging tasks, and conduct ablation experiments to thoroughly examine the performance of our method. The empirical findings suggest that our approach outperforms strong baselines concerning the balance between reward maximization and safety guarantee.

The study offers several improvements to the field, which are enumerated as follows: (1) The introduction of a problem concerning safe RL involving conflicts between reward and cost gradients. (2) The development of a safe RL framework employing soft switching via gradient manipulation. (3) The establishment of a new benchmark for safe RL evaluation, designed to assess the performance of safe RL algorithms; (4) The demonstration that the practical algorithms proposed in this study surpass existing SOTA baselines with respect to both reward and safety performance.

3.2 Related Techniques of Balancing Reward and Safety Optimization

In recent years, numerous safe RL methods have been proposed to ensure RL safety [94]. These safe RL methods can be briefly categorized into three main groups: (1) Control theory-based safe RL: These methods leverage principles from control theory, such as model predictive control and Lyapunov functions, to ensure that the agent operates within safety constraints while learning optimal behavior [128]. (2) Formal methods-based safe RL: These approaches employ formal verification and synthesis techniques, such as temporal logic, to guarantee that the learned policies satisfy safety specifications [80]. (3) Constrained optimization-based safe RL: These methods focus on optimizing the agent's behavior while adhering to safety constraints. Techniques such as constrained policy optimization and Lagrangian relaxation are employed to ensure that the RL algorithm adheres to safety limits throughout the learning process [41].

Compared to the aforementioned methods, constrained optimization-based safe RL methods have gained considerable attention due to their broad applicability. One branch of constrained optimization-based safe RL methods encompasses primal-dual methods [37]. A notable method within this branch is CPO, which employs TRPO [216] in constrained optimization and can nearly guarantee hard constraints via a line search method [184]. PPO-Lagrangian, another representative primal-dual optimization method, is developed based on Lagrangian optimization and dynamically adjusts the Lagrangian multiplier in response to safety violations. Following CPO and PPO-Lagrangian, recent SOTA baselines, such as PCPO [271] and CUP [267], are proposed to ensure learning safety. Another branch of constrained optimization-based safe RL methods consists of primal methods [37]. CRPO [266], a

representative method for primal optimization, directly enhances reward performance while ensuring learning safety within the primal problem. In contrast to primal-dual-based methods, primal-based methods offer ease of implementation and are not burdened by hyperparameter tuning issues related to dual variables. Moreover, primal-based safe RL methods do not necessitate feasible initialization. However, poor initialization can adversely affect the performance of primal-dual optimization-based methods [266].

The methods mentioned above do not explicitly analyze and address the gradient conflicts between reward and cost optimization. This oversight can lead to significantly negative effects on safe RL performance. In contrast to previous works, our proposed method, which is based on primal optimization, necessitates only gradients from the rewards and the costs to ensure safe exploration. This is a key difference from other methods like CRPO, where gradient conflicts may lead to unsafe exploration and wasted samples during training. By focusing on resolving these gradient conflicts, our approach aims to provide a more effective solution for safe RL applications.

3.3 Problem Formulation of Safe RL Balance

In a CMDP $(\mathcal{S}, \mathcal{A}, P, r, \mathbf{c}, \mathbf{b}, \gamma)$, besides the reward function r , we have a cost function $\mathbf{c} = (c_1, \dots, c_n) : \mathcal{S} \times \mathcal{A} \rightarrow \mathbb{R}^n$ and a threshold $\mathbf{b} = (b_1, \dots, b_n) \in \mathbb{R}^n$. In the safety-critical environments, the agent aims at maximizing the expected (discounted) cumulative reward for a given initial distribution ρ while satisfying constraints on the expected (discounted) cumulative cost, i.e.,

$$\max_{\pi \in \Pi} V_r^\pi(\rho), \quad \text{s.t. } V_{c_i}^\pi(\rho) \leq b_i, \quad \forall i = 1, \dots, n. \quad (3.1)$$

where the expectation is taken over all possible trajectories, and $V_r^\pi(\rho)$ and $V_{c_i}^\pi(\rho)$ denote the value function corresponding to the reward and cost functions, respectively.

Primal vs Primal-dual Optimization The current popular policy optimization-based safe RL methods can be generally categorized into the **primal** and **primal-dual** optimization-based approaches.

Primal-Dual Optimization In **primal-dual** optimization, the primal-dual approaches convert the constrained problem equation 3.1 into an unconstrained one by augmenting the objective with a sum of constraints weighted by their corresponding dual variables $\boldsymbol{\lambda}$. The associated Lagrangian function $L(\pi, \boldsymbol{\lambda})$ is defined as:

$$L(\pi, \boldsymbol{\lambda}) := V_r^\pi(\rho) - \boldsymbol{\lambda}^T (V_{\mathbf{c}}^\pi(\rho) - \mathbf{b}), \quad (3.2)$$

where $\boldsymbol{\lambda} \in \mathbb{R}_+^n$. The safe policy is learned from applying a certain policy optimization update, such as (natural) policy gradient alternatively with a gradient descent type update for the dual variables:

$$\pi_{t+1} = \pi_t + \eta_1 \widetilde{\nabla_{\pi} L}(\pi_t, \boldsymbol{\lambda}_t), \quad (3.3)$$

$$\boldsymbol{\lambda}_{t+1} = \boldsymbol{\lambda}_t + \eta_2 (\mathbf{b} - V_{\mathbf{c}}^\pi(\rho))_+, \quad \text{for } t = 0, 1, 2, \dots \quad (3.4)$$

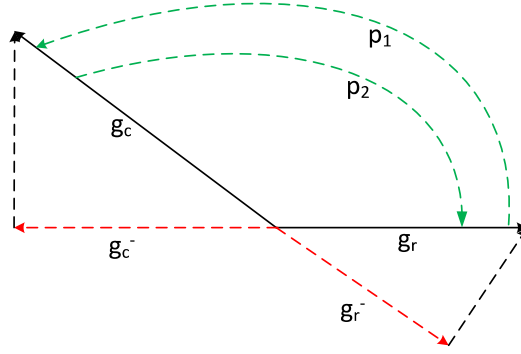


Figure 3.1: Conflicts between reward and cost optimization.

Let $\eta_1 > 0$ denote the policy step-size and $\eta_2 > 0$ represent the dual parameter step-size, and they are used to update π_t and λ_t , respectively. If a safety violation occurs, indicated by $V_c^\pi(\rho) > \mathbf{b}$, then $\lambda^T (V_c^\pi(\rho) - \mathbf{b})$ will increase, encouraging the policy to prioritize safety considerations more strongly. $\nabla_{\pi} L$ can be the policy gradient or its variants, and the dual feasible region $U := [0, C_0]$ is an interval that contains λ^* .

Primal Optimization In **primal** optimization, the necessity for dual variables is eliminated, enabling the immediate optimization of rewards and costs. Such approaches, exemplified by CRPO [266], have demonstrated superior outcomes compared to conventional primal-dual techniques with guaranteed convergence. Nevertheless, when transitioning between reward and cost optimization, conflicting relationships may arise between reward gradients $\mathbf{g}_r$ and cost gradients $\mathbf{g}_c$. This conflict $f_{r_{c_i}}$ has the potential to negatively impact the efficacy of primal methods in terms of both reward and safety performance.

As depicted in Figure 3.1, the reward gradient is represented by $\mathbf{g}_r$, while the cost gradient is denoted by $\mathbf{g}_c$. Additionally, $\mathbf{g}_c^-$ signifies the projection of the cost gradient $\mathbf{g}_c$ onto the plane of the reward gradient $\mathbf{g}_r$, and $\mathbf{g}_r^-$ refers to the projection of the reward gradient $\mathbf{g}_r$ onto the plane of the cost gradient $\mathbf{g}_c$.

During the primal optimization, a transition from cost optimization to reward optimization occurs. In this scenario, the cost gradient optimization process adversely impacts the reward optimization process (p_2). Consequently, the current gradient is expressed as $\mathbf{g} = \mathbf{g}_r - \mathbf{g}_c^-$. Conversely, when switching from reward optimization to cost optimization (p_1), the current gradient is given by $\mathbf{g} = \mathbf{g}_c - \mathbf{g}_r^-$.

Consequently, it is crucial to develop a method for balancing the relation between rewards and costs while simultaneously mitigating the adverse effects of conflicting gradients and minimizing the optimization oscillation on the overall performance. To satisfy the above requirements, we optimize reward and safety performance while minimizing the conflict between reward and cost gradients. This helps prevent conflicting gradients and oscillations during the optimization process.

3.4 Soft Switching Method

In order to tackle the safe RL problem, as illustrated in Equation (3.1), it is imperative to first address the conflict between reward and cost gradients. To this end, we introduce a novel soft switching optimization solution that employs gradient manipulation to achieve a balanced relationship between these gradients. This approach incorporates a slack mechanism designed to optimize both reward and cost. Subsequently, we analyze the gradient change by soft

switching. Then, the convergence analysis is provided. Lastly, we present a safe RL framework though gradient manipulation based on primal optimization, and a practical algorithm that effectively facilitates the implementation of our proposed method in real-world scenarios. Particularly, this soft switching method can also be employed for multi-objective optimization [219]. For example, when certain criteria are met, the policy simultaneously optimizes some objectives. If these criteria are not achieved, the policy shifts its focus exclusively to other objectives.

3.4.1 Soft Switching through Gradient Manipulation

By leveraging the gradient manipulation, we effectively regulate the switching transitions and minimize the oscillations between reward and cost optimization. The objective of soft switching in this context is to enhance the overall efficiency, performance, and reliability of the algorithms, while simultaneously reducing the deviation of gradients between reward and cost components.

3.4.1.1 For Single Constraint

As illustrated in Figure 3.2, the cost gradient of constraint i is denoted by $\mathbf{g}_{c_i}$. For the sake of simplicity, we represent it as $\mathbf{g}_c$. The projection gradient of $\mathbf{g}_c$ on the normal plane of gradient $\mathbf{g}_r$ is given by $\mathbf{g}_{c_i}^+$, while the projection gradient of $\mathbf{g}_r$ on the normal plane of gradient $\mathbf{g}_{c_i}$ is represented by $\mathbf{g}_r^+$. The angle between gradients $\mathbf{g}_r$ and $\mathbf{g}_c$ is denoted by θ .

During the gradient manipulation process, inspired by [279], the gradient projection is employed if the angle θ exceeds 90° , as demonstrated in Equation (3.6) (A simplified illustration of this scenario can be observed in Figure 3.2 (a)), where β_r^+ and β_c^+ denote the weights of gradient $\mathbf{g}_r^+$ and gradient $\mathbf{g}_c^+$, respectively. Conversely, when the angle θ is less than or equal to 90° , Equation (3.7) is leveraged to handle gradient manipulation (A simplified illustration of this scenario can be observed in Figure 3.2 (b)), where β_r and β_c denote the weights of gradient $\mathbf{g}_r$ and gradient $\mathbf{g}_c$, respectively. Our approach aims to minimize optimization oscillations by reducing the deviation between reward and cost gradients, $f_{rc} = f(\mathbf{g}_r, \mathbf{g}_c) = \theta$, particularly for conflicting gradients between reward and cost optimization. This allows us to identify a gradient that can effectively satisfy safety constraints while enhancing reward performance. In the next section, we will analyze how gradient changes with soft switching.

$$\mathbf{g}_r^+ = \mathbf{g}_r - \frac{\mathbf{g}_r \cdot \mathbf{g}_c}{|\mathbf{g}_c|^2} \mathbf{g}_c, \quad \mathbf{g}_c^+ = \mathbf{g}_c - \frac{\mathbf{g}_c \cdot \mathbf{g}_r}{|\mathbf{g}_r|^2} \mathbf{g}_r, \quad (3.5)$$

$$\mathbf{g} = \beta_r^+ \mathbf{g}_r^+ + \beta_c^+ \mathbf{g}_c^+, \quad (3.6)$$

$$\mathbf{g} = \beta_r \mathbf{g}_r + \beta_c \mathbf{g}_c. \quad (3.7)$$

3.4.1.2 For Multiple Constraints

Similar to the single constraint case, in handling multiple constraints, our approach can be effectively applied to compute the cost gradient $\mathbf{g}_c$ and overall gradient $\mathbf{g}$ with respect to multiple constraints ($\mathbf{d}$ is represented as $\mathbf{g}$). As depicted in Figure 3.3 (a), specifically in cases where the angle θ_{c12} between cost gradients $\mathbf{g}_{c1}$ and $\mathbf{g}_{c2}$ is less than 90° , we can employ

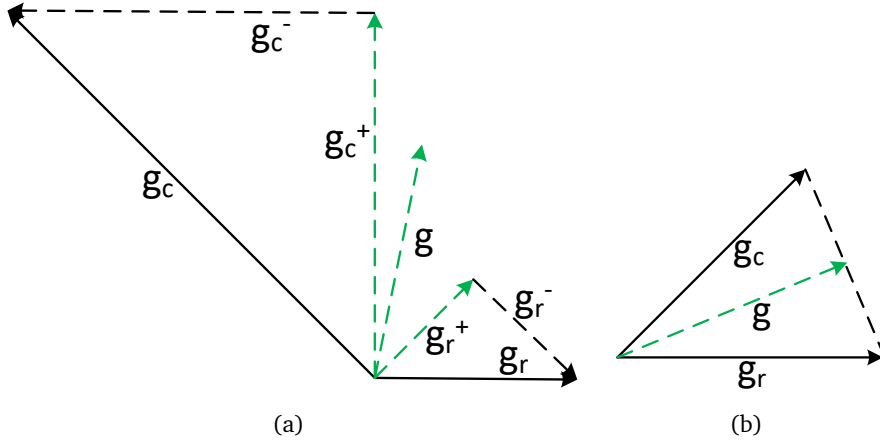


Figure 3.2: Soft switching through gradient manipulation.

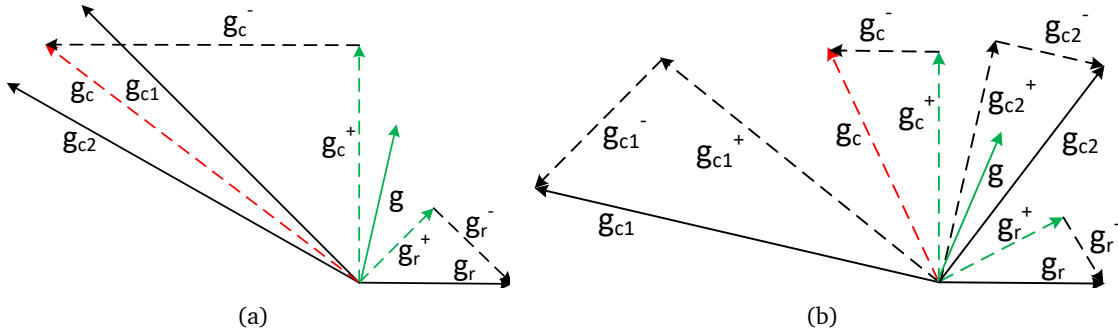


Figure 3.3: Gradient manipulation in multiple constraints.

Equation (16) to calculate the gradient denoted as $\mathbf{g}_c$, as illustrated by the dashed red arrow in the figure. Subsequently, upon obtaining $\mathbf{g}_c$, when the angle θ_{cr} between the cost gradient $\mathbf{g}_c$ and the reward gradient $\mathbf{g}_r$ exceeds 90° , we can utilize Equation (12) to compute the gradient $\mathbf{g}$. This is depicted by the green arrow line in the figure. As illustrated in Figure 3.3 (b), when the angle θ_{c12} between the cost gradients $\mathbf{g}_{c1}$ and $\mathbf{g}_{c2}$ exceeds 90° , we can utilize Equation (12) for the computation of the gradient denoted as $\mathbf{g}_c$. The dashed red arrow visualizes this in the figure. Subsequently, once $\mathbf{g}_c$ has been determined, if the angle θ_{cr} between the cost gradient $\mathbf{g}_c$ and the reward gradient $\mathbf{g}_r$ remains greater than 90° , Equation (12) can once again be employed for the calculation of the gradient $\mathbf{g}$. This is depicted by the green arrow line in the figure.

3.4.2 Gradient Analysis with Soft Switching

In this analysis, for the purpose of simplification, we consider β_r^+ , β_c^+ , β_r and β_c to be equal to 0.5. Other cases follow a similar pattern and can be easily proven based on our analytical framework.

In instances where $\theta \geq 90^\circ$, we have two strategies to handle the conflict gradients between reward and safety optimization. The first strategy is to leverage Equation (3.7) to address the policy gradient, as demonstrated in Figure 3.4 (a), results in the gradient being represented by $\mathbf{g}^-$, which is indicated by the red arrow. The second strategy is to employ Equation (3.6) that allows the gradient to be depicted as $\mathbf{g}$, denoted by the green dashed line in Figure 3.4 (a). To assess which gradient manipulation is better, we provide the following analysis for this instance.

Specifically, by capitalizing on geometric properties, it can be observed that, when $\theta \geq 90^\circ$, the projection of gradient $\mathbf{g}_r$ on the normal plane of gradient $\mathbf{g}_c$ is $\mathbf{g}_r^+$, and the projection of gradient $\mathbf{g}_c$ on the normal plane of gradient $\mathbf{g}_r$ is $\mathbf{g}_c^+$. Consequently, under such conditions, the following property is maintained:

$$\mathbf{g}^- = \frac{\mathbf{g}_r + \mathbf{g}_c}{2} = \frac{\frac{\mathbf{g}_r}{\|\mathbf{g}_r\|} (\|\mathbf{g}_r\|) + \frac{\mathbf{g}_c}{\|\mathbf{g}_c\|} (\|\mathbf{g}_c\|)}{2}, \quad (3.8)$$

$$\cos(\theta) = \frac{\mathbf{g}_r \cdot \mathbf{g}_c}{\|\mathbf{g}_r\| \|\mathbf{g}_c\|}, \quad (3.9)$$

with Equation (3.5) and Equation (3.9), we can observe,

$$\begin{aligned} \mathbf{g} &= \frac{\mathbf{g}_r^+ + \mathbf{g}_c^+}{2} = \frac{\left(\mathbf{g}_r - \frac{\mathbf{g}_r \cdot \mathbf{g}_c}{\|\mathbf{g}_c\|^2} \mathbf{g}_c\right) + \left(\mathbf{g}_c - \frac{\mathbf{g}_c \cdot \mathbf{g}_r}{\|\mathbf{g}_r\|^2} \mathbf{g}_r\right)}{2} \\ &= \frac{\left(\frac{\mathbf{g}_r}{\|\mathbf{g}_r\|} \|\mathbf{g}_r\| - \frac{\cos(\theta) \|\mathbf{g}_r\| \|\mathbf{g}_c\|}{\|\mathbf{g}_c\|^2} \mathbf{g}_c\right) + \left(\frac{\mathbf{g}_c}{\|\mathbf{g}_c\|} \|\mathbf{g}_c\| - \frac{\cos(\theta) \|\mathbf{g}_c\| \|\mathbf{g}_r\|}{\|\mathbf{g}_r\|^2} \mathbf{g}_r\right)}{2}. \end{aligned} \quad (3.10)$$

Under the condition of $\theta \geq 90^\circ$, $\cos(\theta) \leq 0$, we can observe,

$$\begin{aligned} &\left(-\frac{\cos(\theta) \|\mathbf{g}_r\| \|\mathbf{g}_c\|}{\|\mathbf{g}_c\|^2} \mathbf{g}_c\right) + \left(-\frac{\cos(\theta) \|\mathbf{g}_c\| \|\mathbf{g}_r\|}{\|\mathbf{g}_r\|^2} \mathbf{g}_r\right) \geq 0 \\ \Rightarrow \|\mathbf{g}\| &\geq \|\mathbf{g}^-\|. \end{aligned} \quad (3.11)$$

Hence, when $\theta \geq 90^\circ$, the strategy of Equation (3.6) proves to be more effective than the strategy of Equation (3.7) in handling gradient deviations. This indicates that the second strategy can successfully mitigate gradient degradation while addressing conflicting gradients. Specifically, the gradient $\mathbf{g}$ can be considered an equilibrium gradient that strikes a suitable balance between reward optimization and safety constraints. This implies that an increase in the reward or cost expected gradient cannot be achieved by altering its gradient manipulation, given that other gradients remain unmodified.

Under the circumstance where $\theta < 90^\circ$ and $\cos(\theta) > 0$, as illustrated in Figure 3.4 (b), the projection gradients of $\mathbf{g}_r$ on the normal plane of $\mathbf{g}_c$ and $\mathbf{g}_c$ on the normal plane of $\mathbf{g}_r$ yield gradients $\mathbf{g}_r^-$ and $\mathbf{g}_c^-$. Upon observation, it can be noted that,

$$\begin{aligned} \mathbf{g}^- &= \frac{\mathbf{g}_r^- + \mathbf{g}_c^-}{2} \\ &= \frac{\left(\frac{\mathbf{g}_r}{\|\mathbf{g}_r\|} \|\mathbf{g}_r\| - \frac{\cos(\theta) \|\mathbf{g}_r\| \|\mathbf{g}_c\|}{\|\mathbf{g}_c\|^2} \mathbf{g}_c\right) + \left(\frac{\mathbf{g}_c}{\|\mathbf{g}_c\|} \|\mathbf{g}_c\| - \frac{\cos(\theta) \|\mathbf{g}_c\| \|\mathbf{g}_r\|}{\|\mathbf{g}_r\|^2} \mathbf{g}_r\right)}{2}, \end{aligned} \quad (3.12)$$

with $\theta < 90^\circ$, $\cos(\theta) > 0$, the following property holds,

$$-\left(\frac{\cos(\theta) \|\mathbf{g}_r\| \|\mathbf{g}_c\|}{\|\mathbf{g}_c\|^2}\right) - \left(\frac{\cos(\theta) \|\mathbf{g}_c\| \|\mathbf{g}_r\|}{\|\mathbf{g}_r\|^2}\right) < 0, \quad (3.13)$$

$$\mathbf{g} = \frac{\mathbf{g}_r + \mathbf{g}_c}{2} = \frac{\frac{\mathbf{g}_r}{\|\mathbf{g}_r\|} \|\mathbf{g}_r\| + \frac{\mathbf{g}_c}{\|\mathbf{g}_c\|} \|\mathbf{g}_c\|}{2}. \quad (3.14)$$

Thus, we can observe $\|\mathbf{g}\| > \|\mathbf{g}^-\|$. In this example, $\theta < 90^\circ$, the gradient managed by the second strategy, as illustrated in Equation (3.6), can be observed in Equation(3.12). Concurrently, the gradient addressed using the first strategy, as depicted in Equation(3.7), is presented in Equation (3.14). Upon examination, it becomes evident that under these conditions, the first strategy surpasses the second strategy in effectively handling deviations in

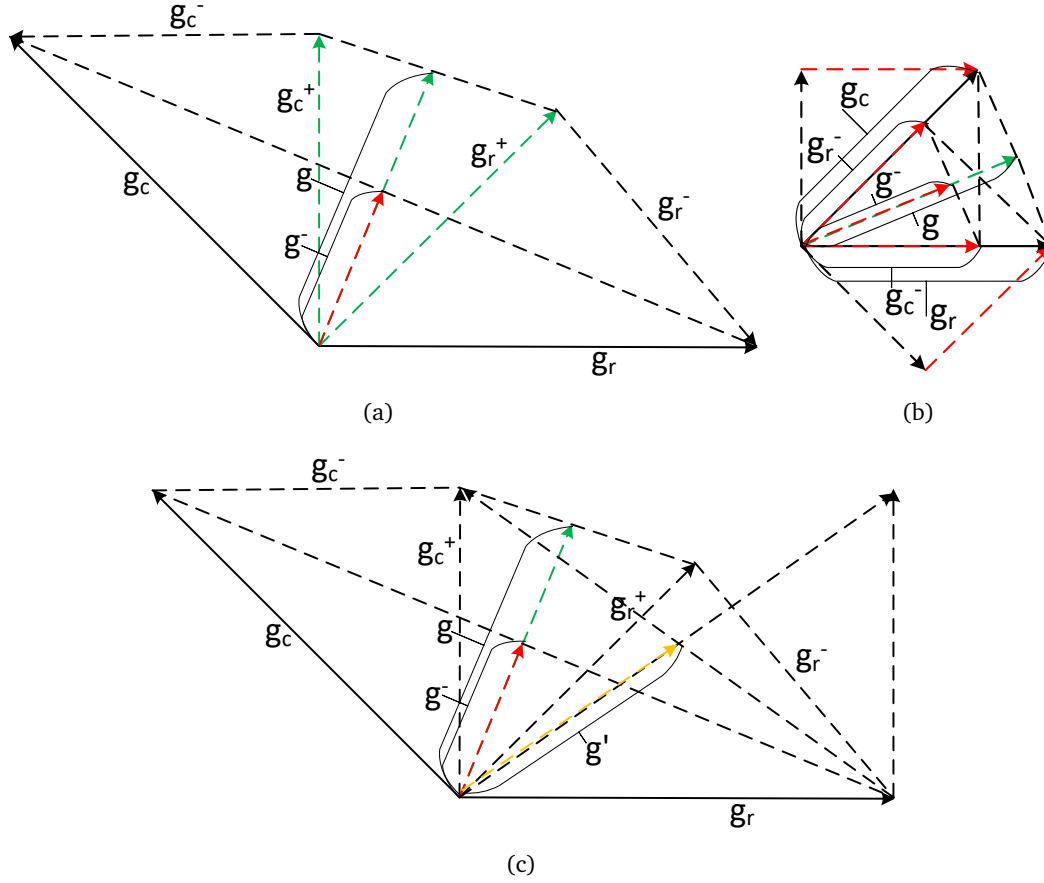


Figure 3.4: Analysis of Soft switching through gradient manipulation.

reward and cost gradients. Furthermore, the first strategy is capable of mitigating gradient degradation while simultaneously reducing gradient deviation.

In single constraint and multiple constraint cases, the gradient projection manipulation used in this study is inspired by [279], and we further leverage it for reward and safety balance. As illustrated in Figure 3.4 (c), the yellow dashed line signifies the updated gradient, $\mathbf{g}'$, generated by Algorithm 1 of gradient surgery [279], the angle, θ , between $\mathbf{g}'$ and the original cost gradient, $\mathbf{g}_c$, remains greater than 90° . This observation implies that the relation between $\mathbf{g}'$ and $\mathbf{g}_c$ continues to exhibit a conflicting nature, potentially resulting in inadequate handling of the original cost gradient. Based on the subsequent analysis as shown in Equation (3.15), our gradient manipulation for reward and safety balance demonstrates improved performance. In the upcoming experiment section, we also provide ablation experiments to investigate the effectiveness of different gradient manipulation methods.

$$\begin{aligned}
 \mathbf{g}' &= \frac{\mathbf{g}_r + \mathbf{g}_c^+}{2} = \frac{\mathbf{g}_r + \left(\mathbf{g}_c - \frac{\mathbf{g}_c \cdot \mathbf{g}_r}{\|\mathbf{g}_r\|^2} \mathbf{g}_r \right)}{2}, \\
 &= \frac{\frac{\mathbf{g}_r}{\|\mathbf{g}_r\|} \|\mathbf{g}_r\| + \left(\frac{\mathbf{g}_c}{\|\mathbf{g}_c\|} \|\mathbf{g}_c\| - \frac{\cos(\theta) \|\mathbf{g}_c\| \|\mathbf{g}_r\|}{\|\mathbf{g}_r\|^2} \mathbf{g}_r \right)}{2}, \\
 &\Rightarrow \|\mathbf{g}\| > \|\mathbf{g}'\|.
 \end{aligned} \tag{3.15}$$

3.4.3 A Framework for Safe Reinforcement Learning with Soft Switching

In this section, we present a comprehensive framework referred to as PCRPO, which iteratively optimizes performance until convergence is achieved. As demonstrated in Algorithm 1, we

propose a novel approach with slack techniques to address the deviation of reward and cost gradients, particularly for conflicting gradients.

Algorithm 1 PCRPO: A Framework of Projection Constraint-Soft-Rectified Policy Optimization

```

1: Inputs: initial policy with parameters  $\pi_{w_0}$ , positive slack value  $h_t^+$ , negative slack value  $h_t^-$ , the cost value of constraint  $i$  as  $V_{c_{i,t}}^{\pi_{w_0}}(\rho)$  at step  $t$ , the cost limit of constraint  $i$  as  $b_i$ .
2: for  $t = 0, \dots, T - 1$  do
3:   Policy evaluation under  $\pi_{w_t}$  involves estimating the values of rewards and constraints.
4:   Sample pairs  $(s_j, a_j)$  from the buffer  $\mathcal{B}_t$  according to the distribution  $\rho \cdot \pi_{w_t}$  and compute the estimation  $V_{r,t}^{\pi_{w_t}}(\rho)$  and  $V_{c_{i,t}}^{\pi_{w_t}}(\rho)$ , where  $s_j$  represents the state and  $a_j$  represents the action,  $j$  is the index for the sampled pairs.
5:   if  $h_t^+ = +\infty, h_t^- = 0$  then
6:     if For any  $i, b_i > V_{c_{i,t}}^{\pi_{w_t}}(\rho)$  then
7:       Update policy  $\pi_{w_t}$  to maximize reward value  $V_{r,t}^{\pi_{w_t}}(\rho)$  with Equation (3.16).
8:     else
9:       Compute the update direction  $\mathbf{d}$  with Equation (3.21), and update the projection policy leveraging the obtained  $\mathbf{d}$  and Equation (3.19).
10:    end if
11:   else if  $h_t^+ = 0, h_t^- = -\infty$  then
12:     if For any  $i, b_i > V_{c_{i,t}}^{\pi_{w_t}}(\rho)$  then
13:       Compute the update direction  $\mathbf{d}$  with Equation (3.21), and update the projection policy leveraging the obtained  $\mathbf{d}$  and Equation (3.19).
14:     else
15:       Update policy to minimize cost  $V_{c_{i,t}}^{\pi_{w_t}}(\rho)$  and ensure safety with Equation (3.17).
16:     end if
17:   else if  $+\infty > h_t^+ > 0, 0 > h_t^- > -\infty$  then
18:     if  $h^+$  adaptive decreases then
19:        $h_t^+ \leftarrow h_t^+ - h_t^+/T$ 
20:     end if
21:     if  $h^+$  adaptive increases then
22:        $h_t^- \leftarrow h_t^- - h_t^-/T$ 
23:     end if
24:     if For any  $i, V_{c_{i,t}}^{\pi_{w_t}}(\rho) > (h_t^+ + b_i)$  then
25:       Choose any unsatisfied constraint  $i_t$  and update policy to ensure safety with Equation (3.17).
26:     else if For any  $i, (h_t^- + b_i) < V_{c_{i,t}}^{\pi_{w_t}}(\rho) < (h_t^+ + b_i)$  then
27:       Compute the update direction  $\mathbf{d}$  with Equation (3.21), and update the projection policy leveraging the obtained  $\mathbf{d}$  and Equation (3.19).
28:     else if For any  $i, V_{c_{i,t}}^{\pi_{w_t}}(\rho) < (h_t^- + b_i)$  then
29:       Update policy  $\pi_{w_t}$  to maximize reward  $V_{r,t}^{\pi_{w_t}}(\rho)$  with Equation (3.16).
30:     end if
31:   end if
32: end for
33: Outputs:  $\pi_{w_t}$ .

```

Case one: In the event that the slack value tends toward infinity, i.e., $h^+ \rightarrow +\infty$ and $h^- = 0$, the optimization process is adapted based on the satisfaction of safety constraints. When a safety violation occurs, the optimization exclusively focuses on safety by employing Equation (3.17), where w is the parameters represented by neural networks, η is the step size

of gradient update. Conversely, if safety constraints are satisfied, the optimization process incorporates the projection gradient, as delineated in Equation (3.19).

Case two: In the event that the slack value is denoted by $h^+ = 0$ and $h^- \rightarrow -\infty$, a safety violation necessitates the enhancement of the reward and concurrent reduction of cost by employing Equation (3.19). Conversely, when safety requirements are fulfilled, the focus shifts solely to the optimization of reward performance, as demonstrated by Equation (3.16).

Case three: In situations where the slack value is confined to the range of $+\infty > h^+ > 0$ and $0 > h^- > -\infty$, several circumstances can be observed. If upper slack, lower slack, and safety violations occur simultaneously, the optimization process is devoted solely to addressing safety concerns, as indicated by Equation (3.17). In the absence of upper slack violations, while lower slack and safety violations transpire concurrently, the strategy involves enhancing the reward and concurrently reducing the cost by employing Equation (3.19). Conversely, when upper slack and safety violations are not present, but a lower slack violation persists, the same approach of augmenting the reward and minimizing the cost is implemented using Equation (3.19). Finally, in the absence of violations related to upper slack, lower slack, and safety, the primary focus is directed towards optimizing reward performance, as demonstrated by Equation (3.16).

$$w_{t+1} = w_t + \eta \bar{\Delta}_t^r, \mathbf{g}^r = \bar{\Delta}_t^r, \quad (3.16)$$

$$w_{t+1} = w_t - \eta \bar{\Delta}_t^{c_i}, \mathbf{g}^c = -\bar{\Delta}_t^{c_i}, \quad (3.17)$$

where from Lemma 5.1 of [6], we have

$$\bar{\Delta}_t^r = (1 - \gamma)^{-1} \bar{Q}_t^r(s, a), \bar{\Delta}_t^{c_i} = (1 - \gamma)^{-1} \bar{Q}_t^{c_i}(s, a). \quad (3.18)$$

$$w_{t+1} = w_t + \eta \cdot \left(\frac{\mathbf{g}_r^+ + \mathbf{g}_c^+}{2} \right), w_{t+1} = w_t + \eta \cdot \left(\frac{\mathbf{g}_r + \mathbf{g}_c}{2} \right). \quad (3.19)$$

Inspired by CRPO [266], we implement our algorithm within the context of the primal optimization setting. Similarly, we initially evaluate the policy and subsequently improve it while addressing safety constraints.

3.4.3.1 Policy Evaluation of Safe RL Balance

During the policy evaluation step, the objective is to learn Q-functions that accurately evaluate the previous policy π_t . To accomplish this, we train distinct Q-functions for both reward and constraints.

$$Q_{i,k+1}^{\pi_w}(s, a) = Q_{i,k}^{\pi_w} + \ell_k [r_i(s, a) + \gamma Q_{i,k}^{\pi_w}(s', a') - Q_{i,k}^{\pi_w}(s, a)], \quad (3.20)$$

where $s \sim \mu_{\pi_w}$, $a \sim \pi_w(s)$, $s' \sim P(\cdot | s, a)$, $a' \sim \pi_w(s')$, ℓ_k is the learning rate and i denotes the reward or any of the constraints. $\bar{Q}_i(s, a)$ can be estimated via $Q_{i,K_{TD}}^{\pi_w}(s, a)$, where K_{TD} is the iteration number of using TD learning methods.

3.4.3.2 Policy Improvement for Reward and Safety

The policy gradient [235] of the reward value function $f_r(\pi_w)$ has been derived as $\nabla f_r(\pi_w) = \mathbb{E}[Q_r^{\pi_w}(s, a) \phi_w(s, a)]$, where $\phi_w(s, a) := \nabla_w \log \pi_w(a | s)$ is the score function. Similarly, for the value function of cost i , we have $\nabla f_{c_i}(\pi_w) = \mathbb{E}[Q_{c_i}^{\pi_w}(s, a) \phi_w(s, a)]$.

In scenarios where the optimization of both reward and safety i is desired, it is necessary to select a non-conflicting gradient descent $\mathbf{d}$ on the natural gradients of reward and cost

gradients. This selection aims to optimize reward and safety individually, subject to the constraint that the KL divergence between the updated and previous policy remains below a specified threshold.

$$\mathbf{d} = \frac{\mathbf{g}_r^+ + \mathbf{g}_{c_i}^+}{2} \text{ or } \frac{\mathbf{g}_r + \mathbf{g}_{c_i}}{2}. \quad (3.21)$$

3.4.3.3 Correlation-Reduction for Stochastic Gradient Manipulation

In practical applications, the challenge of acquiring imprecise policy gradient feedback is frequently encountered. This imprecision stems from the restricted number of sampled trajectories employed to estimate $Q_r^{\pi_{w_t}}$ or $Q_{c_i}^{\pi_{w_t}}$, subsequently introducing stochastic noise into the system. The study conducted in [292] has revealed that, within a stochastic setting, conventional gradient manipulation techniques may fail to converge to an optimal solution.

Let the weight factors $\lambda t = (\lambda_t^r, \lambda_t^{c_1}, \dots, \lambda_t^{c_n})$ be represented as $\mathbf{d} = \lambda_t^r \mathbf{g}_r + \lambda_t^{c_i} \mathbf{g}_{c_i}$ for the time step t . The primary cause of this convergence failure lies in the substantial correlation between the weight factors λt and the stochastic gradients, resulting in a biased composite gradient. To address this issue within the context of the PCRPO framework, we concentrate on the specific conditions that ensure the variance of the natural policy gradient estimator progressively approaches zero. One possible approach to achieve this involves utilizing TD learning in equation 4.3 to estimate $Q^{\pi_{w_t}} i$, assuming K_{TD} is adequately large.

3.4.4 Comparison to CRPO

Compared to CRPO [266], a primal safe RL algorithm, our algorithm exhibits two distinct differences: the addition of upper and lower slack values to the constraint thresholds, and the approach we take to optimize the policy concerning the balance between reward and safety performance. CRPO focuses on optimizing for reward, only shifting to safety optimization if a safety constraint is hard violated. This can lead to a back-and-forth between reward and cost optimizations, particularly when constraints are near their threshold boundaries. To circumvent such oscillations and to prevent any performance degradation in other objectives, we employ a projected gradient descent approach. This ensures a balanced and efficient way of handling both reward and safety concerns.

3.4.5 Convergence Analysis of Safe Reinforcement Learning Balance

For the iterates π_{w_t} manipulated using our proposed method, **we can guarantee performance monotonic improvement and convergence to the optimal performance in cases where $180^\circ > \theta \geq 0^\circ$** . Our theorem enables the derivation of other settings in a straightforward manner.

Theorem 1. *Under the assumption of Lipschitz continuity with a constant L , for the iterates π_{w_t} generated through our gradient manipulation, when $180^\circ > \theta \geq 90^\circ$, as depicted in Equation(3.22), both the upper and lower bounds of performance updates can be observed. As demonstrated in Equation (3.23), our method can guarantee performance monotonic improvement and converge to the optimal performance.*

$$\begin{aligned}
& \frac{1}{L} \cdot \left(\frac{5\|\mathbf{g}_r\|^2 + 5\|\mathbf{g}_c\|^2 - 5\cos(\theta)\|\mathbf{g}_r\|^2 - 5\cos(\theta)\|\mathbf{g}_c\|^2 + \cos^2(\theta) - \cos(\theta)\|\mathbf{g}_r\|\|\mathbf{g}_c\|}{8} \right) \geq \\
& f(w_{t+1}) - f(w_t) \geq \\
& \eta \cdot \left(\frac{3\|\mathbf{g}_r\|^2 + 3\|\mathbf{g}_c\|^2 - 3\cos^2(\theta)\|\mathbf{g}_r\|^2 - 3\cos^2(\theta)\|\mathbf{g}_c\|^2 - 2\cos^3(\theta)\|\mathbf{g}_r\|\|\mathbf{g}_c\| + 2\cos(\theta)\|\mathbf{g}_r\|\|\mathbf{g}_c\|}{8} \right). \tag{3.22}
\end{aligned}$$

In the case of $180^\circ > \theta \geq 90^\circ$, the following property holds,

$$\begin{aligned}
& 3\|\mathbf{g}_r\|^2 + 3\|\mathbf{g}_c\|^2 - 2\cos^3(\theta)\|\mathbf{g}_r\|\|\mathbf{g}_c\| > 3\cos^2(\theta)\|\mathbf{g}_r\|^2 + 3\cos^2(\theta)\|\mathbf{g}_c\|^2 - 2\cos(\theta)\|\mathbf{g}_r\|\|\mathbf{g}_c\| \\
& \implies f(w_{t+1}) - f(w_t) > 0. \tag{3.23}
\end{aligned}$$

Proof. When $\theta \geq 90^\circ$, under the assumption that the gradient function, $\nabla f(w)$, exhibits Lipschitz continuity with a constant L , it can be deduced that the difference between the Hessian matrix $\nabla^2 f(w)$ and the scaled identity matrix LI is a negative semi-definite matrix [293]. By leveraging this property, one can conduct a quadratic expansion of the function $f(w)$ in the vicinity of $f(w)$, which subsequently leads to the derivation of the following inequality:

$$\begin{aligned}
f(w_{t+1}) & \geq f(w_t) + \nabla f(w_t)^T (w_{t+1} - w_t) - \frac{1}{2}L \|w_{t+1} - w_t\|^2, \\
f(w_{t+1}) & \leq f(w_t) + \nabla f(w_t)^T (w_{t+1} - w_t) + \frac{1}{2}L \|w_{t+1} - w_t\|^2. \tag{3.24}
\end{aligned}$$

With Lipschitz continuity, the assumption holds,

$$\eta \leq \frac{1}{L}. \tag{3.25}$$

Leveraging gradient update functions (3.16) and (3.17). For the case, $\theta \geq 90^\circ$, with Equations (3.5), (3.6) and (3.9), we have the performance improvement upper bound:

$$\begin{aligned}
f(w_{t+1}) &\leq f(w_t) + \nabla f(w_t)^T (w_{t+1} - w_t) + \frac{1}{2}L \|w_{t+1} - w_t\|^2 \\
&= f(w_t) + (\mathbf{g}_r + \mathbf{g}_c)^T \left(\eta \cdot \left(\frac{\mathbf{g}_r^+ + \mathbf{g}_c^+}{2} \right) \right) + \frac{1}{2}L \left\| \eta \cdot \left(\frac{\mathbf{g}_r^+ + \mathbf{g}_c^+}{2} \right) \right\|^2 \\
&= f(w_t) + (\mathbf{g}_r + \mathbf{g}_c)^T \eta \cdot \left(\frac{\left(\mathbf{g}_r - \frac{\mathbf{g}_r \cdot \mathbf{g}_c}{\|\mathbf{g}_c\|^2} \mathbf{g}_c \right) + \left(\mathbf{g}_c - \frac{\mathbf{g}_c \cdot \mathbf{g}_r}{\|\mathbf{g}_r\|^2} \mathbf{g}_r \right)}{2} \right) \\
&\quad + \frac{1}{2}L \left\| \eta \cdot \left(\frac{\left(\mathbf{g}_r - \frac{\mathbf{g}_r \cdot \mathbf{g}_c}{\|\mathbf{g}_c\|^2} \mathbf{g}_c \right) + \left(\mathbf{g}_c - \frac{\mathbf{g}_c \cdot \mathbf{g}_r}{\|\mathbf{g}_r\|^2} \mathbf{g}_r \right)}{2} \right) \right\|^2 \\
&= f(w_t) + \eta \cdot \left(\frac{\|\mathbf{g}_r\|^2 + \|\mathbf{g}_c\|^2 - \cos^2(\theta)\|\mathbf{g}_r\|^2 - \cos^2(\theta)\|\mathbf{g}_c\|^2}{2} \right) \\
&\quad + \frac{1}{2}L\eta^2 \cdot \left(\frac{\|\mathbf{g}_r\|^2 + \|\mathbf{g}_c\|^2 - \cos^2(\theta)\|\mathbf{g}_r\|^2 - \cos^2(\theta)\|\mathbf{g}_c\|^2 + 2\cos^3(\theta)\|\mathbf{g}_r\|\|\mathbf{g}_c\| - 2\cos(\theta)\|\mathbf{g}_r\|\|\mathbf{g}_c\|}{4} \right) \\
&\leq f(w_t) + \\
&\quad \frac{1}{L} \cdot \left(\frac{5\|\mathbf{g}_r\|^2 + 5\|\mathbf{g}_c\|^2 - 5\cos^2(\theta)\|\mathbf{g}_r\|^2 - 5\cos^2(\theta)\|\mathbf{g}_c\|^2 + 2\cos^3(\theta)\|\mathbf{g}_r\|\|\mathbf{g}_c\| - 2\cos(\theta)\|\mathbf{g}_r\|\|\mathbf{g}_c\|}{8} \right) \\
&\implies f(w_{t+1}) - f(w_t) \leq \\
&\quad \frac{1}{L} \cdot \left(\frac{5\|\mathbf{g}_r\|^2 + 5\|\mathbf{g}_c\|^2 - 5\cos^2(\theta)\|\mathbf{g}_r\|^2 - 5\cos^2(\theta)\|\mathbf{g}_c\|^2 + 2\cos^3(\theta)\|\mathbf{g}_r\|\|\mathbf{g}_c\| - 2\cos(\theta)\|\mathbf{g}_r\|\|\mathbf{g}_c\|}{8} \right) \\
&\geq 0.
\end{aligned} \tag{3.26}$$

Similarly, we can have the lower bound,

$$\begin{aligned}
f(w_{t+1}) - f(w_t) &\geq \\
&\eta \cdot \left(\frac{3\|\mathbf{g}_r\|^2 + 3\|\mathbf{g}_c\|^2 - 3\cos^2(\theta)\|\mathbf{g}_r\|^2 - 3\cos^2(\theta)\|\mathbf{g}_c\|^2 - 2\cos^3(\theta)\|\mathbf{g}_r\|\|\mathbf{g}_c\| + 2\cos(\theta)\|\mathbf{g}_r\|\|\mathbf{g}_c\|}{8} \right).
\end{aligned} \tag{3.27}$$

Thus, performance updates at each iteration t can be bounded as follows,

$$\begin{aligned}
&\frac{1}{L} \cdot \left(\frac{5\|\mathbf{g}_r\|^2 + 5\|\mathbf{g}_c\|^2 - 5\cos(\theta)\|\mathbf{g}_r\|^2 - 5\cos(\theta)\|\mathbf{g}_c\|^2 + \cos^2(\theta) - \cos(\theta)\|\mathbf{g}_r\|\|\mathbf{g}_c\|}{8} \right) \geq \\
&f(w_{t+1}) - f(w_t) \geq \\
&\eta \cdot \left(\frac{3\|\mathbf{g}_r\|^2 + 3\|\mathbf{g}_c\|^2 - 3\cos^2(\theta)\|\mathbf{g}_r\|^2 - 3\cos^2(\theta)\|\mathbf{g}_c\|^2 - 2\cos^3(\theta)\|\mathbf{g}_r\|\|\mathbf{g}_c\| + 2\cos(\theta)\|\mathbf{g}_r\|\|\mathbf{g}_c\|}{8} \right).
\end{aligned} \tag{3.28}$$

When $90^\circ \leq \theta < 180^\circ$, the following property holds,

$$\begin{aligned}
& 3(1 - \cos^2(\theta))\|\mathbf{g}_r\|^2 + 3(1 - \cos^2(\theta))\|\mathbf{g}_c\|^2 > 2\cos(\theta)(\cos^2(\theta) - 1)\|\mathbf{g}_r\|\|\mathbf{g}_c\| \\
& 3\sin^2(\theta)\|\mathbf{g}_r\|^2 + 3\sin^2(\theta)\|\mathbf{g}_c\|^2 > -2\cos(\theta)\sin^2(\theta)\|\mathbf{g}_r\|\|\mathbf{g}_c\| \\
& 3\|\mathbf{g}_r\|^2 + 3\|\mathbf{g}_c\|^2 > -2\cos(\theta)\|\mathbf{g}_r\|\|\mathbf{g}_c\|, \text{ if } \sin(\theta) \neq 0 \\
& \frac{\|\mathbf{g}_r\|}{\|\mathbf{g}_c\|} + \frac{\|\mathbf{g}_c\|}{\|\mathbf{g}_r\|} > 1 > -\frac{2}{3}\cos(\theta), \\
& \implies 3\|\mathbf{g}_r\|^2 + 3\|\mathbf{g}_c\|^2 - 3\cos^2(\theta)\|\mathbf{g}_r\|^2 - 3\cos^2(\theta)\|\mathbf{g}_c\|^2 - 2\cos^3(\theta)\|\mathbf{g}_r\|\|\mathbf{g}_c\| + 2\cos(\theta)\|\mathbf{g}_r\|\|\mathbf{g}_c\| > 0, \\
& \implies f(w_{t+1}) - f(w_t) > 0.
\end{aligned} \tag{3.29}$$

This implies that, through our gradient manipulation, in instances where $180^\circ > \theta \geq 90^\circ$, the proposed method can guarantee monotonic performance improvement. Furthermore, the method demonstrates the ability to converge towards optimal performance. $\square$

Theorem 2. *Under the assumption of Lipschitz continuity with a constant L , for the iterates π_{w_t} generated through our gradient manipulation, in the case of $90^\circ > \theta \geq 0^\circ$, as depicted in Equation(3.30), both the upper and lower bounds of performance updates can be observed. As demonstrated in Equation (3.31), our method can guarantee performance monotonic improvement and converge to the optimal performance.*

$$\begin{aligned}
& \frac{1}{L} \cdot \left(\frac{5\|\mathbf{g}_r\|^2 + 10\cos(\theta)\|\mathbf{g}_r\|\|\mathbf{g}_c\| + 5\|\mathbf{g}_c\|^2}{8} \right) \geq f(w_{t+1}) - f(w_t) \\
& \geq \eta \cdot \left(\frac{3\|\mathbf{g}_r\|^2 + 6\cos(\theta)\|\mathbf{g}_r\|\|\mathbf{g}_c\| + 3\|\mathbf{g}_c\|^2}{8} \right)
\end{aligned} \tag{3.30}$$

$$\begin{aligned}
& \left(\frac{3\|\mathbf{g}_r\|^2 + 6\cos(\theta)\|\mathbf{g}_r\|\|\mathbf{g}_c\| + 3\|\mathbf{g}_c\|^2}{8} \right) > 0 \\
& \implies f(w_{t+1}) - f(w_t) > 0,
\end{aligned} \tag{3.31}$$

Proof. Similar to Theorem 2, the upper bound is observed,

$$\begin{aligned}
f(w_{t+1}) & \leq f(w_t) + \nabla f(w_t)^T (w_{t+1} - w_t) + \frac{1}{2}L \|w_{t+1} - w_t\|^2 \\
& = f(w_t) + (\mathbf{g}_r + \mathbf{g}_c)^T \left(\eta \cdot \left(\frac{\mathbf{g}_r + \mathbf{g}_c}{2} \right) \right) + \frac{1}{2}L \left\| \eta \cdot \left(\frac{\mathbf{g}_r + \mathbf{g}_c}{2} \right) \right\|^2 \\
& = f(w_t) + \eta \cdot \left(\frac{\|\mathbf{g}_r\|^2 + 2\cos(\theta)\|\mathbf{g}_r\|\|\mathbf{g}_c\| + \|\mathbf{g}_c\|^2}{2} \right) \\
& \quad + L\eta^2 \cdot \left(\frac{\|\mathbf{g}_r\|^2 + 2\cos(\theta)\|\mathbf{g}_r\|\|\mathbf{g}_c\| + \|\mathbf{g}_c\|^2}{8} \right) \\
& \leq f(w_t) + \frac{1}{L} \cdot \left(\frac{\|\mathbf{g}_r\|^2 + 2\cos(\theta)\|\mathbf{g}_r\|\|\mathbf{g}_c\| + \|\mathbf{g}_c\|^2}{2} \right) \\
& \quad + \frac{1}{L} \cdot \left(\frac{\|\mathbf{g}_r\|^2 + 2\cos(\theta)\|\mathbf{g}_r\|\|\mathbf{g}_c\| + \|\mathbf{g}_c\|^2}{8} \right) \\
& = f(w_t) + \frac{1}{L} \cdot \left(\frac{5\|\mathbf{g}_r\|^2 + 10\cos(\theta)\|\mathbf{g}_r\|\|\mathbf{g}_c\| + 5\|\mathbf{g}_c\|^2}{8} \right).
\end{aligned} \tag{3.32}$$

Similarly, we can have the lower bound,

$$\begin{aligned}
f(w_{t+1}) &\geq f(w_t) + \nabla f(w_t)^T (w_{t+1} - w_t) - \frac{1}{2}L \|w_{t+1} - w_t\|^2 \\
&= f(w_t) + (\mathbf{g}_r + \mathbf{g}_c)^T \left(\eta \cdot \left(\frac{\mathbf{g}_r + \mathbf{g}_c}{2} \right) \right) - \frac{1}{2}L \left\| \eta \cdot \left(\frac{\mathbf{g}_r + \mathbf{g}_c}{2} \right) \right\|^2 \\
&= f(w_t) + \eta \cdot \left(\frac{\|\mathbf{g}_r\|^2 + 2\cos(\theta)\|\mathbf{g}_r\|\|\mathbf{g}_c\| + \|\mathbf{g}_c\|^2}{2} \right) \\
&\quad - L\eta^2 \cdot \left(\frac{\|\mathbf{g}_r\|^2 + 2\cos(\theta)\|\mathbf{g}_r\|\|\mathbf{g}_c\| + \|\mathbf{g}_c\|^2}{8} \right) \\
&\geq f(w_t) + \eta \cdot \left(\frac{\|\mathbf{g}_r\|^2 + 2\cos(\theta)\|\mathbf{g}_r\|\|\mathbf{g}_c\| + \|\mathbf{g}_c\|^2}{2} \right) \\
&\quad - \eta \cdot \left(\frac{\|\mathbf{g}_r\|^2 + 2\cos(\theta)\|\mathbf{g}_r\|\|\mathbf{g}_c\| + \|\mathbf{g}_c\|^2}{8} \right) \\
&= f(w_t) + \eta \cdot \left(\frac{3\|\mathbf{g}_r\|^2 + 6\cos(\theta)\|\mathbf{g}_r\|\|\mathbf{g}_c\| + 3\|\mathbf{g}_c\|^2}{8} \right).
\end{aligned} \tag{3.33}$$

Thus, in this case of $90^\circ > \theta \geq 0^\circ$, at each iteration t , we can have the performance update bound,

$$\begin{aligned}
\frac{1}{L} \cdot \left(\frac{5\|\mathbf{g}_r\|^2 + 10\cos(\theta)\|\mathbf{g}_r\|\|\mathbf{g}_c\| + 5\|\mathbf{g}_c\|^2}{8} \right) &\geq f(w_{t+1}) - f(w_t) \\
&\geq \eta \cdot \left(\frac{3\|\mathbf{g}_r\|^2 + 6\cos(\theta)\|\mathbf{g}_r\|\|\mathbf{g}_c\| + 3\|\mathbf{g}_c\|^2}{8} \right).
\end{aligned} \tag{3.34}$$

In this case, $\cos(\theta) \geq 0$, we can have

$$\begin{aligned}
\left(\frac{3\|\mathbf{g}_r\|^2 + 6\cos(\theta)\|\mathbf{g}_r\|\|\mathbf{g}_c\| + 3\|\mathbf{g}_c\|^2}{8} \right) &> 0 \\
\implies f(w_{t+1}) - f(w_t) &> 0,
\end{aligned} \tag{3.35}$$

This implies that the proposed method ensures a monotonic enhancement in performance, ultimately converging to the optimal value. $\square$

3.5 Experiments of Safe RL Balance

In the experimental section, we investigate the constraint satisfaction of policies trained using our proposed method and compare its performance with SOTA safe RL algorithms. Employing the developed benchmark, *Safety-MuJoCo Benchmark*, we first compare our approach with a representative primal optimization-based safe RL method, CRPO [266], a strong baseline established in 2021. CRPO demonstrates superior performance in comparison to several SOTA baselines such as PDO [208].

To further emphasize the effectiveness of our method, we implement our algorithm on a popular benchmark, *Omnisafe* [113], and compare it with several representative primal-dual optimization-based safe RL methods. These methods include SOTA baselines, such as PCPO [271], CUP [267], and PPO Lagrangian (PPOlag) [113]. By contrasting our approach with these established methods, we aim to demonstrate the potential advantages and improvements that our proposed method offers in terms of safety and efficiency for RL applications with safety constraints.

In particular, our proposed method belongs to the category of primal optimization-based safe RL approaches, thereby avoiding the challenges associated with hyperparameter tuning related to dual variables. Additionally, our method does not necessitate feasible initialization, in contrast to some primal-dual optimization-based methods where poor initialization can adversely impact performance [266]. By circumventing these challenges, our proposed method aims to provide a more reliable and efficient solution for ensuring safety in RL applications. Additionally, we conduct ablation studies to investigate the impact of diverse cost limits and sensitivity to slack bounds.

3.5.1 Experiments on *Safety-MuJoCo Benchmark*

We have developed a *Safety-MuJoCo Benchmark*¹ based on MuJoCo [134, 244] to evaluate the performance of safe RL algorithms. This benchmark differs from traditional safe RL benchmarks, such as *Omnisafe*² [113] that is developed based on Safety Gym³ [208]. In *Omnisafe*, the cost constraint is set as the velocity limit, and the reward is determined by the speed at which the robot runs. Conversely, our benchmark considers not only velocity constraints but also the health of the robot. For example, whether the robot falls or whether its joints exceed the limit values of motion control are taken into account. For a comprehensive description of these considerations, please refer to Appendix C. As depicted in Figure 3.5 (a) and (b), our method demonstrates remarkably superior performance in comparison to CRPO with respect to both reward maximization and safety preservation. Similarly, Figure 3.5 (c) and (d) illustrate our method’s significant improvement over CRPO in terms of reward and cost performance.

3.5.2 Experiments on *Omnisafe Benchmark*

We implement our algorithm on *Omnisafe* and compare its performance with several SOTA baselines within the *Omnisafe* framework. The safety constraint is set as a constant threshold, where if the agent moves at a higher velocity than this threshold, it incurs a cost of 1 per time step. We test our method, PCRPO, alongside PCPO, CUP, and PPOLag methods on various environments, including Hopper and Ant.

As illustrated in Figure 3.6 (a) and (b), on the SafetyHopperVelocity-v1 task, our algorithm exhibits superior reward performance compared to SOTA baselines and maintains reliable safety. In contrast, SOTA baselines such as CUP and PPOLag struggle to ensure safety, and their reward performance is worse than our algorithm. Notably, our approach outperforms PCPO in both reward and safety performance. In Figure 3.6 (c) and (d), our algorithm effectively ensures complete safety on the SafetyAntVelocity-v1 task while achieving comparable reward performance. Specifically, our algorithm demonstrates greater safety than CUP and PPOLag, which can not ensure safety on the task. While PCPO can also ensure safety, its reward performance is inferior to our algorithm. Furthermore, our algorithm demonstrates faster convergence than the baselines.

3.5.3 Ablation Experiments

¹<https://github.com/SafeRL-Lab/Safety-MuJoCo.git>

²<https://github.com/PKU-Alignment/omnisafe.git>

³<https://github.com/openai/safety-gym.git>

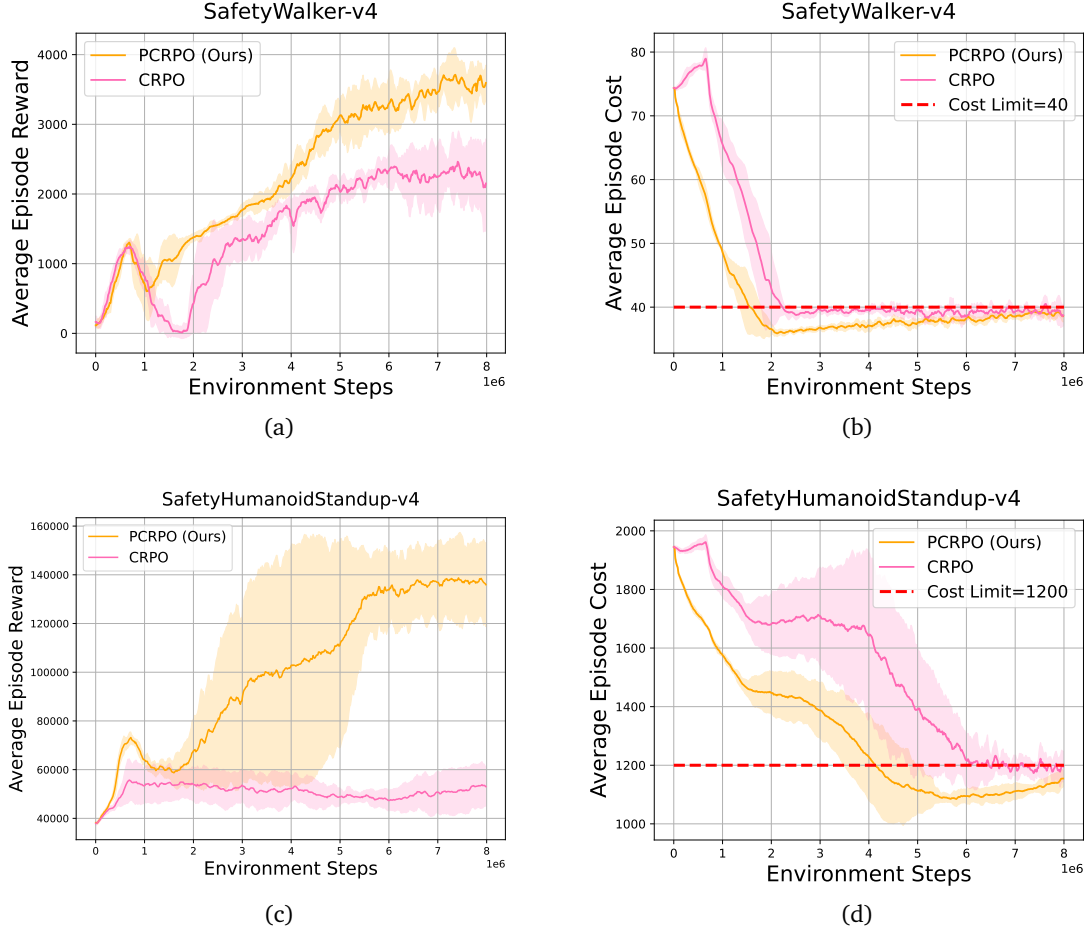


Figure 3.5: Compared with CRPO on the SafetyWalker and SafetyHumanoidStandup Tasks. To encourage more learning exploration, we initiate the optimization of safety after 640000 steps.

3.5.3.1 Ablation Experiments of Slack Settings

As illustrated in Figures 3.7 (a) and (b), we perform an ablation study on various slack settings. PCRPO-2SR represents $h_i^+ \rightarrow +\infty, h_i^- = 0$, where we primarily optimize reward while slightly ensuring safety. PCRPO-3SR-G denotes $h_i^+ = 20, h_i^- = 0$, with h_i^+ gradually decreasing to zero as the number of iteration steps increases. In this setting, we aim to optimize reward and safety simultaneously when $(b_i + h_i^+) > C_i > b_i$. PCRPO-4S-F corresponds to $h_i^+ = 20, h_i^- = -20$, where we optimize safety and reward at the static slack boundary. As the experiment results show, our algorithm's cost value converges to the boundary. PCRPO-4S-G represents $h_i^+ = 20, h_i^- = -20$, with h_i^+ and h_i^- gradually decreasing to zero as the number of iteration steps increases. In this setting, we optimize safety and reward at the dynamic slack boundary, and as demonstrated by the experimental results, our algorithm's cost value converges to the cost limit while maintaining good reward performance. Notably, all our algorithms demonstrate superior results compared to CRPO in terms of balancing reward and safety optimization. This experimental setup allows us to analyze the impact of various slack configurations on the performance of our method, providing insights into the balance between reward and safety optimization.

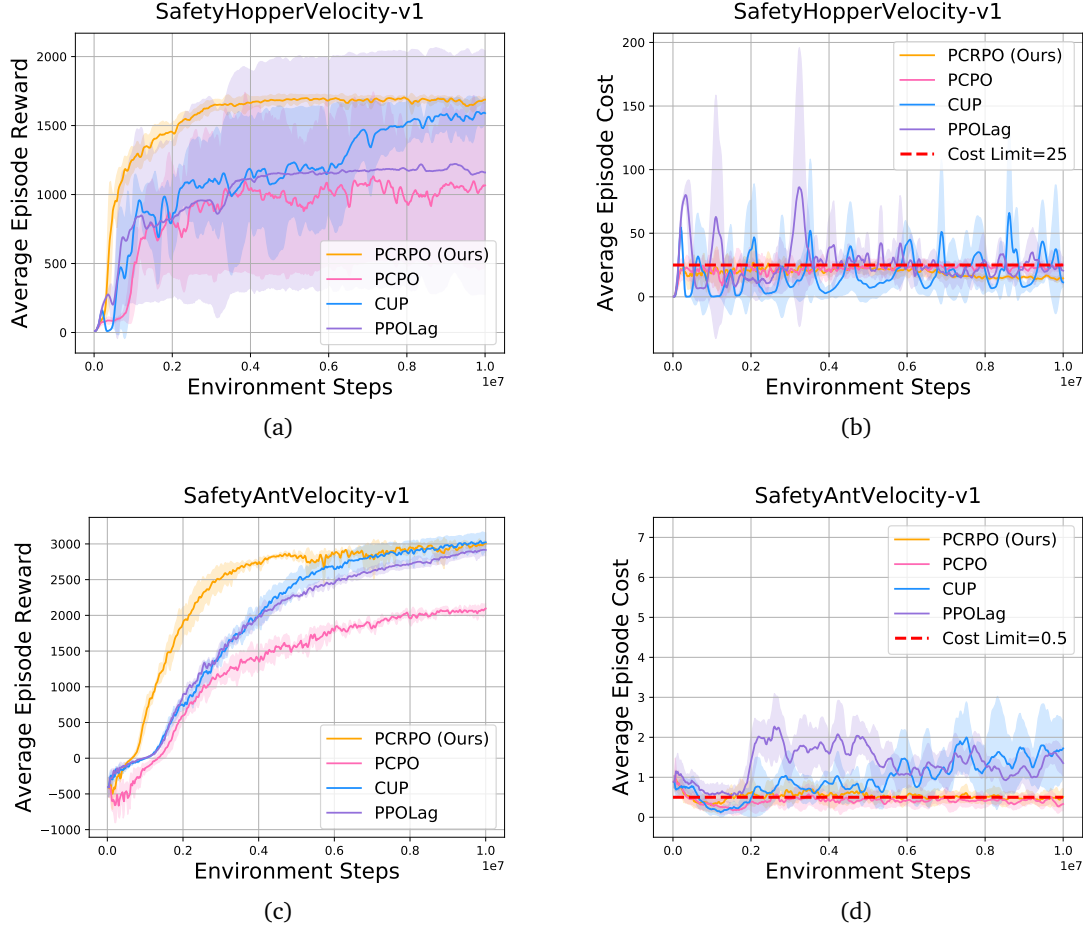


Figure 3.6: Compared with PCPO, CUP, PPOLag baselines on SafetyHopperVelocity-v1 and SafetyAntVelocity-v1 tasks.

3.5.3.2 Ablation Experiments of Gradient Manipulation Methods

As depicted in Figures 3.7 (c) and (d), we employ the gradient manipulation technique as outlined in Algorithm 1 of gradient surgery [279] for learning safety, named as SCRPO. The experimental results demonstrate that PCRPO outperforms SCRPO in terms of safety and reward performance. These findings further demonstrate the consistency of our theoretical analysis presented in the gradient analysis section.

3.6 Summary of Safe RL Balance

In this study, we address the issue of gradient conflicts between reward and cost by employing gradient manipulation. Specifically, we first propose a novel solution called PCRPO, which incorporates soft switching to balance reward and safety optimization in safe RL. Moreover, a slack technique is developed to help alleviate the conflict between reward and safety optimization. Our theoretical analysis demonstrates that our method can guarantee performance monotonic improvement while also analyzing the upper and lower bounds of the performance update. Then, we evaluate the effectiveness of our method using the *Safety-MuJoCo Benchmark* that we developed, as well as a popular safe RL benchmark, *Omnisafe*. Finally, the experimental results show that our method outperforms the strong baselines, indicating its

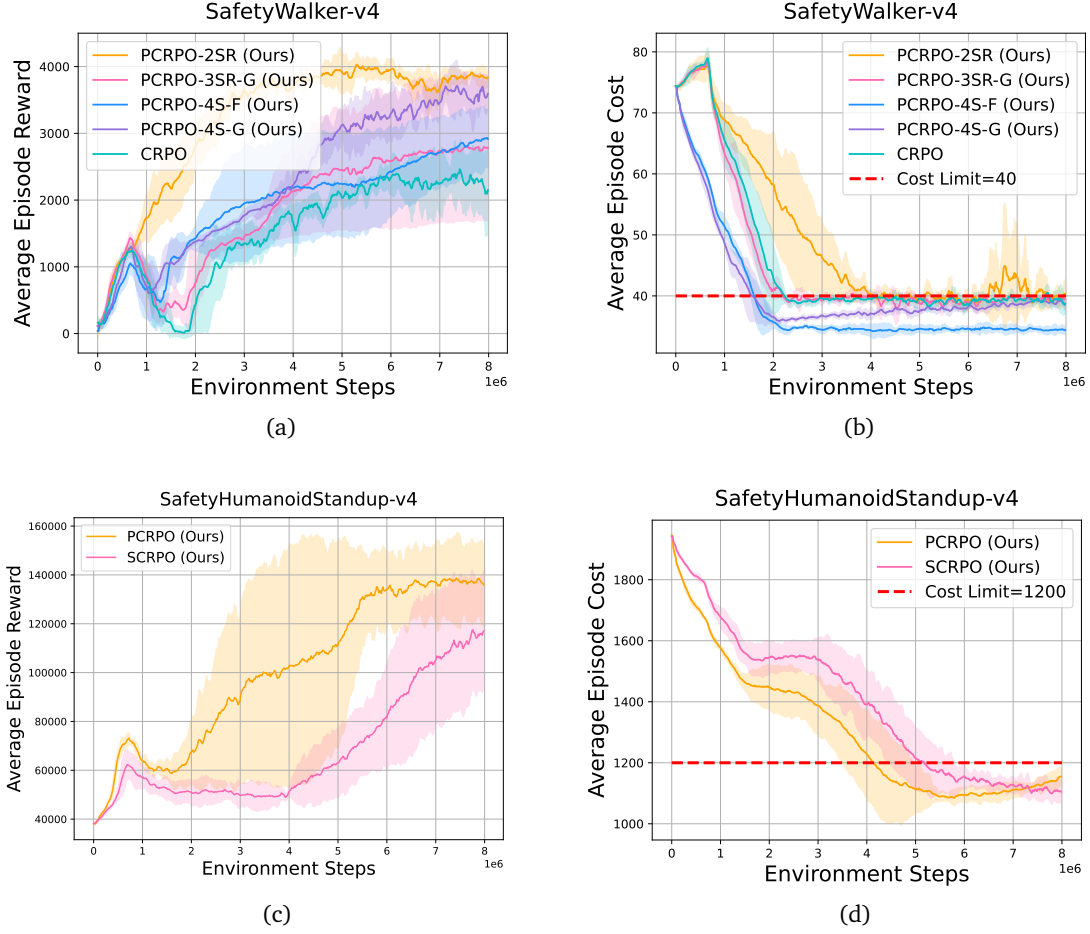


Figure 3.7: (a & b) Ablation experiments of different slack settings on the SafetyWalker task. (c & d) Ablation experiments of different gradient manipulation methods on the SafetyHumanoidStandup task. To encourage more learning exploration, we initiate the safety optimization after 640000 steps on the SafetyWalker task.

superior performance in addressing the challenges associated with safe RL.

3.7 Experiment Details of Safe Reinforcement Learning Balance

3.7.1 Additional Experiments

As illustrated in Table 3.1, we conducted an experiment by modifying the learning rate (lr) values of all baselines to 0.001, disabling the linear learning rate decay (*linear lr decay*) for all baselines, and maintaining other parameters as specified in Table 3.1. The results of this experiment are depicted in Figure 3.8, where our algorithm’s performance is significantly superior to that of the SOTA baselines. In particular, as illustrated in Figures 3.8 (a-d), we conducted experiments on the SafetyAntVelocity tasks with identical hyperparameters. To better showcase the results, Figures 3.8 (a) and (b) display the outcomes of PCRPO and PCPO, while Figures 3.8 (c) and (d) present the results of CUP and PPOLag. These findings reveal that CUP and PPOLag do not perform well under the same conditions. Figures 3.8 (e) and (f) depict the results of all algorithms on the SafetyHopperVelocity tasks. The outcomes demonstrate that our algorithm not only achieves superior reward performance and faster

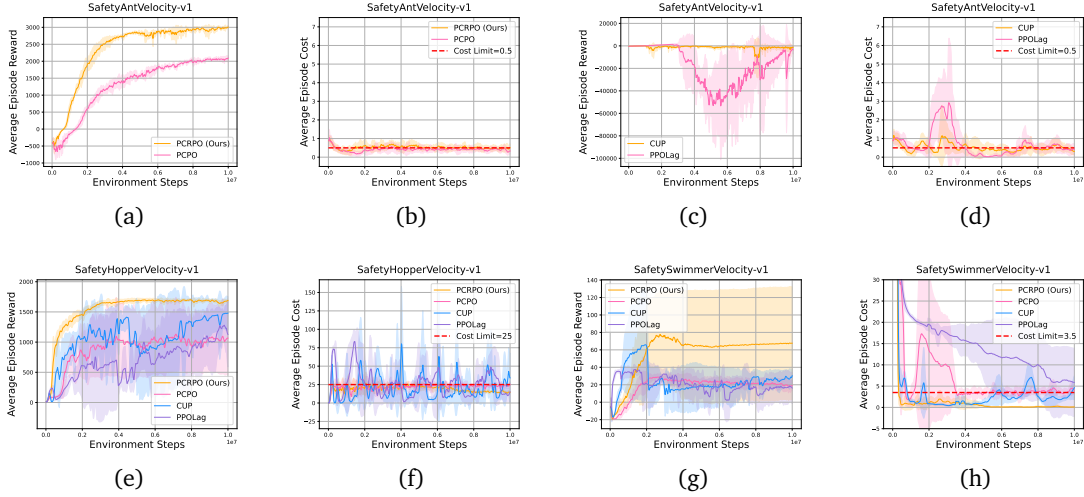


Figure 3.8: In the additional experiments, the performance of the proposed method was compared with several baseline algorithms, including PCPO [271], CUP [267], and PPOLag [113]. The comparison was conducted on three safety-constrained tasks: SafetyAntVelocity-v1, SafetyHopperVelocity-v1, and SafetySwimmerVelocity-v1.

convergence compared to all SOTA baselines, but also ensures safety, which the SOTA baselines fail to guarantee. Similar observations can be made for the SafetySwimmerVelocity tasks, as shown in Figures 3.8 (g) and (h).

It is worth noting that the parameters of the other baselines were fine-tuned by [113], which may represent the best performance achievable by the SOTA baselines, and our method can outperform all the baselines in terms of reward, safety performance and convergence. By thoroughly evaluating the effectiveness and efficiency of our proposed method in comparison to the baseline algorithms, we aimed to provide valuable insights into their respective strengths and weaknesses, as well as identify potential areas for improvement and future research directions.

3.7.2 Implementation Details

In the conducted experiments, the primary hyperparameters employed can be observed in Tables 3.1 and 3.2. Additionally, the cost limit configurations are presented in Table 3.3, while the slack settings are demonstrated in Table 3.4. To conduct the experiments, a server equipped with 40 CPU cores (Intel® Xeon(R) Gold 5218R CPU @ 2.10GHz $\times$ 80) and 1 GTX-970 GPU (NVIDIA GeForce GTX 970/PCIe/SSE2) is utilized. The operating system running on the server is Ubuntu 18.04.

values parameters	algorithms	CUP	PCPO	PPOLag	PCRPO
device		cpu	cpu	cpu	cpu
torch threads		1	1	1	1
vector env nums		1	1	1	1
parallel		1	1	1	1
total steps		10 million	10 million	10 million	10 million
steps per epoch		20000	20000	20000	20000
update iters		40	10	40	10
batch size		64	128	64	128
target kl		0.01	0.01	0.02	0.01
entropy coef		0	0	0	0
reward normalize		False	False	False	False
cost normalize		False	False	False	False
obs normalize		True	True	True	True
use max grad norm		True	True	True	True
max grad norm		40	40	40	40
use critic norm		True	True	True	True
critic norm coef		0.001	0.001	0.001	0.001
gamma		0.99	0.99	0.99	0.99
cost gamma		0.99	0.99	0.99	0.99
lam		0.95	0.95	0.95	0.95
lam c		0.95	0.95	0.95	0.95
clip		0.2	\	0.2	\
adv estimation method		gae	gae	gae	gae
standardized rew adv		True	True	True	True
standardized cost adv		True	True	True	True
penalty coef		0	0	0	0
cg damping		\	0.1	\	0.1
cg iters		\	15	\	15
actor type		gaussian learning	gaussian learning	gaussian learning	gaussian learning
hidden sizes		[64, 64]	[64, 64]	[64, 64]	[64, 64]
activation		tanh	tanh	tanh	tanh
lr		0.0003	0.001	0.0003	0.001
lagrangian multiplier init		0.001	\	0.001	\
lambda lr		0.035	\	0.035	\

Table 3.1: Key hyparameters used in Omnisafe.

Parameters	value	Parameters	value
gamma	0.995	tau	0.97
l2-reg	1e-3	kl	0.05
damping	1e-1	batch-size	16000
epoch	500	episode length	1000
grad-c	0.5	neural network	MLP
hidden layer dim	64	accept ratio	0.1
energy weight	1.0	forward reward weight	1.0

Table 3.2: Key hyparameters used in Safety-MuJoCo Benchmark.

Environment	Cost limit
SafetyWalker-v4	40
SafetyHumanoidStandup-v4	1200
SafetyAntVelocity-v1	0.5
SafetyHopperVelocity-v1	25
SafetySwimmerVelocity-v1	3.5
SafetySwimmerVelocity-v1 (ablation of cost limit)	0.08
SafetyWalker-v4 (ablation of slack settings)	40
SafetyHumanoidStandup-v4 (ablation of gradient manipulation)	1200

Table 3.3: Cost limit settings in experiments.

Environment	Slack value
SafetyWalker-v4	$h^+ = 5, h^- = -5$
SafetyHumanoidStandup-v4	$h^+ = 300, h^- = -300$
SafetyAntVelocity-v1	$h^+ = 0.25, h^- = -0.25$
SafetyHopperVelocity-v1	$h^+ = 0, h^- = -9$
SafetySwimmerVelocity-v1	$h^+ = 0, h^- \rightarrow \infty$
SafetySwimmerVelocity-v1 (ablation of cost limit)	$h^+ = 0.04, h^- = -0.04$
SafetyWalker-v4 (ablation of slack settings)	$h^+ \rightarrow +\infty, h^- = 0$ (PCRPO-2SR)
SafetyWalker-v4 (ablation of slack settings)	$h^+ = 20, h^- = 0$ (PCRPO-3SR-G)
SafetyWalker-v4 (ablation of slack settings)	$h^+ = 20, h^- = -20$ (PCRPO-4S-F & PCRPO-4S-G)
SafetyHumanoidStandup-v4 (ablation of gradient manipulation methods)	$h^+ = 300, h^- = -300$ (PCRPO & SCRPO)

Table 3.4: Slack settings in experiments.

4

Safe Multi-Objective Robot Reinforcement Learning

In numerous RL problems involving safety-critical systems, a key challenge lies in balancing multiple objectives while simultaneously meeting all safety constraints. To tackle this issue, we propose a primal-based framework that can handle policy optimization between multi-objective reward learning and constraint adherence. Our method employs a novel natural policy gradient manipulation method to optimize multiple RL objectives and balance conflicting gradients between different objectives, since the current simple weighted average gradient direction may not be beneficial for specific objectives due to misaligned gradients of different objectives. Particularly, We establish theoretical convergence and constraint violation guarantees, and our proposed method also outperforms prior SOTA methods on challenging safe multi-objective RL tasks.

4.1 Introduction of Safe Multi-Objective Robot Reinforcement Learning

RL is widely investigated in single objective domains, e.g., robotics [89, 126], autonomous driving [86, 125], large language model [188], and finance [44]. However, a significant challenge arises when a policy must address multiple objectives within a single task or manage multiple tasks concurrently. Direct optimization of scalarized objectives can lead to suboptimal performance, with the optimizer often struggling to make progress, resulting in a considerable decline in learning performance [250]. A significant cause of this issue is the phenomenon of conflicting gradients [279]. Here, gradients associated with different objectives may vary in scale, potentially leading the largest gradient to dominate the update. Moreover, they might point in different directions, i.e., $\nabla f_i(\pi)^\top \nabla f_j(\pi) < 0, i \neq j, i, j \in [m] = \{1, \dots, m\}$, causing the performance of one objective to deteriorate during the optimization of another. While recent studies have shown that linear scalarization can be competitive [133], it may fall short when faced with safety-critical constraints. Indeed, ensuring the safe application of RL algorithms in real-world settings, especially those dealing with multiple objectives, is paramount [107]. This study seeks to answer the key question:

How can we manage multi-reward optimization while ensuring safety constraints?

Addressing this problem, akin to a multi-dimensional tug of war, requires nuance. Each objective is a team pulling in its own direction, yet confined by the boundaries of safety—a balancing act of objectives and safety. Inspired by this dynamic, we devise a comprehensive framework for Constrained Multi-Objective RL (CMORL) using gradient manipulation and

constraint rectification. It operates in three stages: (1) Estimating Q-functions from the existing policy. (2) If all constraints are satisfactorily met, the policy is updated via the manipulated natural policy gradient (NPG) of multiple objectives to minimize the gradient conflicts. (3) If not, the policy is updated following the NPG of the unsatisfied constraint. These steps are iteratively repeated until convergence is achieved.

Accompanying this framework, we provide a theoretical analysis, including convergence analysis and violation guarantee analysis. Using the insights from this analysis, we develop a practical algorithm to manage multi-objective RL while ensuring safety during learning. We further deploy our algorithm on safe multi-objective tasks in the MuJoCo environment [244] and compare our method with the SOTA safe baseline, CRPO [266], and SOTA safe multi-objective RL methods, such as LP3 [107]. Our experimental results suggest that our method outperforms CRPO and LP3 in striking a balance between reward performance and safety violation.

Our study offers several significant contributions to the field of safe multi-objective RL, which are delineated as follows: (1) A novel framework for safe multi-objective RL, wherein a comprehensive analysis of both theoretical convergence and constraint violation guarantees is conducted. (2) The development of a benchmark grounded in MuJoCo environments (Named *Safe Multi-Objective MuJoCo*), aimed at scrutinizing the efficacy of safe multi-objective learning. (3) The superior performance by our proposed method in terms of striking a balance between safety concerns and the accomplishment of multiple reward objectives, as evidenced across numerous challenging tasks within the realm of safe multi-objective RL.

4.2 Related Techniques of Safe Multi-Objective Reinforcement Learning

In recent years, numerous methods are proposed to help deploy RL in real-world applications [88, 89, 107, 261], which try to solve the safe exploration problem [94] or satisfy multi-objective requirements during RL exploration [251] from the perspective of safe or multi-objective RL.

Safe Reinforcement Learning. Safe RL has gained significant attention as it helps address learning safety problems during RL deployment in real-world applications. Safe RL can be considered as a constrained optimization problem [94]. For example, several safe RL methods leverage Gaussian Processes to model the safe state space during exploration [25, 231, 247, 254]. In contrast to modeling the safe state, some safe RL methods attempt to search for a safe policy within the constrained action space [51, 52, 80, 128, 142, 168], e.g., based on formal methods, the exploration action is verified via temporal logic verification during exploration [142]. Furthermore, by optimizing the average cumulative cost of each trajectory, several constrained policy optimization-based methods are proposed, such as CPO[5], PCPO[271], RCPO[241], and CRPO[266].

Multi-Objective Reinforcement Learning (MORL). There are two settings in MORL [107]. The first involves a single policy in multi-objective optimization, while the second involves a multi-policy set that satisfies multi-objective requirements. Most MORL methods are developed based on the first setting, where a single policy needs to meet multiple objective conditions simultaneously [2]. Additionally, various multi-objective learning methods are proposed to optimize policy performance, such as multi-objective learning as a bargaining game [182], Cagrad [147], the Multiple-Gradient Descent Algorithm (MGDA)[65], and PCGrad [279]. Methods in the second setting attempt to learn a complete set of Pareto frontiers and use a posterior selection to satisfy multi-objective requirements [249]. Examples include MORL optimization based on manifold space to find better solutions on the Pareto frontier [191, 192].

The difference between Safe RL and MORL: Safe RL focuses specifically on the agent’s safety. There are numerous methods to achieve this; for instance, after modeling a safe state or safe action space, the problem reduces to searching for a single objective. Consequently, safe RL can be treated as a single-objective learning problem. In contrast, MORL considers the trade-offs among different objectives and needs to account for the performance with respect to each. While MORL can be employed to address safe RL problems, it necessitates the consideration of constraints and cannot simply minimize or maximize each objective independently. The optimization of other objectives must be carried out under the premise of ensuring safety. For example, in safe RL, a problem arises where it is important to first satisfy safety constraints and then optimize other objectives without compromising safety performance. This issue cannot be resolved by straightforwardly applying standard MORL methods. It is imperative to consider how to maximize other objectives while guaranteeing safety first.

The methods mentioned above address RL safety or multi-objective requirements separately without considering both aspects simultaneously. Our focus is on achieving safe MORL, which involves ensuring exploration safety in multi-objective RL settings. The most similar work to ours is the Learning Preferences and Policies in Parallel (LP3) algorithm [107], which is proposed based on the Multi-Objective Maximum Posterior Policy optimization (MO-MPO) [2]. In this approach, a supervised learning algorithm is used to learn preferences, and then a policy is trained based on Lagrangian optimization. However, their method heavily depends on Q-estimation, which may not accurately represent safe preferences; the gradient conflict between each objective is not analyzed, and neither convergence analysis nor safety violation guarantees are provided. In contrast to LP3 [107], we propose a primal-based framework that can balance policy optimization between multi-objective learning and constraint satisfaction based on conflict-averse NPG. In our approach, the conflict gradient is analyzed between each objective performance, and convergence analysis and safety violation guarantees are provided based on gradient manipulation and constraint rectification.

4.3 Preliminaries and Problem Formulation of Safe Multi-Objective Reinforcement Learning

4.3.1 Multi-Objective Reinforcement Learning

A MORL is a tuple $(\mathcal{S}, \mathcal{A}, \{r_i\}_{i=1}^m, P, \rho, \gamma)$, where $\mathcal{S}$ and $\mathcal{A}$ are state and action spaces; $r_i : \mathcal{S} \times \mathcal{A} \rightarrow [0, r_{\max}]$ is the reward function; $m \geq 2$ denotes the number of objectives; $P : \mathcal{S} \times \mathcal{A} \times \mathcal{S} \rightarrow [0, 1]$ is the transition kernel, with $P(s' | s, a)$ denoting the probability of transitioning to state s' from previous state s given action a ; $\rho : \mathcal{S} \rightarrow [0, 1]$ is the initial state distribution; and $\gamma \in (0, 1)$ is the discount factor. A policy $\pi \in \Pi : \mathcal{S} \rightarrow \mathcal{P}(\mathcal{A})$ is a mapping from the state space to the space of probability distributions over the actions, with $\pi(\cdot | s)$ denoting the probability of selecting action a in state s . When the associated Markov chain $P(s' | s) = \sum_{\mathcal{A}} P(s' | s, a) \pi(a | s)$ is ergodic, we denote μ_π as the stationary distribution of this MDP, i.e. $\int_{\mathcal{S}} P(s' | s) \mu_\pi(ds) = \mu_\pi(s')$. Moreover, we define the visitation measure induced by the policy π as $v_\pi(s, a) = (1 - \gamma) \sum_{t=0}^{\infty} \gamma^t P(s_t = s, a_t = a)$.

For a given policy π and a reward function r_i , we define the state value function as $V_i^\pi(s) = \mathbb{E}[\sum_{t=0}^{\infty} \gamma^t r_i(s_t, a_t) | s_0 = s, \pi]$, the state-action value function as

$$Q_i^\pi(s, a) = \mathbb{E}[\sum_{t=0}^{\infty} \gamma^t r_i(s_t, a_t) | s_0 = s, a_0 = a, \pi],$$

the advantage function as $A_i^\pi(s, a) = Q_i^\pi(s, a) - V_i^\pi(s)$, and the expected total reward function

$$f_i(\pi) = \mathbb{E} \left[\sum_{t=0}^{\infty} \gamma^t r_i(s_t, a_t) \right] = \mathbb{E}_{\rho} [V_i^{\pi}(s)] = \mathbb{E}_{\rho \cdot \pi} [Q_i^{\pi}(s, a)].$$

In MORL, we aim to find a single optimal policy that maximizes multiple expected total reward functions simultaneously, termed as

$$\max_{\pi \in \Pi} F(\pi) = (f_1(\pi), \dots, f_m(\pi))^{\top}. \quad (4.1)$$

4.3.2 Constrained Multi-Objective Reinforcement Learning

The CMORL problem refers to a formulation of MORL that involves additional *hard constraints* that restrict the allowable policies. The constraints take the form of costs that the agent may incur when taking actions at certain states, denoted by the functions $r_{m+1}, \dots, r_{m+p}$. Each of these cost functions maps a tuple (s, a) to a corresponding cost value. The function $f_{m+i}(\pi)$ represents the expected total cost incurred by the agent with respect to cost function r_{m+i} . The objective of the agent in CMORL is to solve a multi-objective RL problem subject to the aforementioned hard constraints:

$$\max_{\pi \in \Pi} F(\pi), \text{ s.t. } f_i(\pi) \leq c_i, \forall i = m+1, \dots, m+p, \quad (4.2)$$

where c_i is a fixed limit for the i -th constraint. We define the safety set

$$\Pi_{\text{safe}} = \{\pi \in \Pi \mid f_i(\pi) \leq c_i, \forall i = m+1, \dots, m+p\},$$

and the optimal policy $\pi^* = \arg \max_{\pi \in \Pi_{\text{safe}}} F(\pi)$ for CMORL in equation 4.2. In practice, a convenient way to solve RL is to parameterize the policy and then iteratively optimize the policy over the parameter space. Let $\{\pi_w : \mathcal{S} \rightarrow \mathcal{P}(\mathcal{A}) \mid w \in \mathcal{W}\}$ be a parameterized policy class, where $\mathcal{W}$ is the parameter space. Then, the problem in equation 4.2 can be written as

$$\max_{w \in \mathcal{W}} F(\pi_w), \text{ s.t. } f_i(\pi_w) \leq c_i, \forall i = m+1, \dots, m+p.$$

In CMORL, we extend the notion of the Pareto frontier, which is defined to compare the policies, from the unconstrained MDP [292] to the safety-constrained MDP.

Definition 1 (Safe Pareto Frontier). *For any two policies $\pi, \pi' \in \Pi$, we say that π dominates π' if $f_i(\pi) \leq f_i(\pi')$ for all i , and there exists one i such that $f_i(\pi) < f_i(\pi')$; otherwise, we say that π does not dominate π' . A solution $\pi^* \in \Pi_{\text{safe}}$ is called safe Pareto optimal if it is not dominated by any other safe policy in Π_{safe} . The set of all safe Pareto optimal policies is the safe Pareto frontier.*

The primary aim of CMORL is to identify a safe Pareto optimal policy. Nevertheless, the concurrent learning of multiple objectives introduces a complex optimization problem, as it entails the consideration of numerous objectives simultaneously. This complexity arises from the need to effectively balance trade-offs between conflicting objectives while maintaining safety constraints throughout the learning process.[250]. The most popular multi-objective formulation in practice is the linear scalarization of all objectives given relative preferences for each objective $\xi_i, i \in [m]$:

$$\max_{w \in \mathcal{W}} \xi^{\top} F(\pi_w), \text{ s.t. } f_i(\pi_w) \leq c_i, \forall i = m+1, \dots, m+p.$$

Even when this linear scalarization formulation gives exactly the true objective, directly optimizing it could lead to undesirable performance due to conflicting gradients, dominating gradients, and high curvature [279].

In this paper, we aim to find a safe Pareto optimal solution using the gradient-based method by starting from an arbitrary initialization policy π_t and iteratively finding the next policy

π_{t+1} by moving against a direction $\mathbf{d}_t$ with step size η , i.e., $\pi_{t+1} = \pi_t + \eta \mathbf{d}_t$. The design of the direction $\mathbf{d}_t$ is the key to the success of CMORL. A good direction $\mathbf{d}_t$ should enable us to move from a policy π_t to π_{t+1} such that either π_{t+1} dominates π_t or π_{t+1} improves the hard constraint satisfaction compared with π_t , or both.

4.4 Constraint-Rectified Multi-Objective Policy Optimization

In this section, we introduce a general framework called CR-MOPO which decomposes safe Pareto optimal policy learning into three sub-problems and iterates until convergence:

1. Policy evaluation: estimate Q-functions given the current policy.
2. Policy improvement for the multi-objectives: update policy based on the manipulated NPG of multi-objectives when constraints are all approximately satisfied.
3. Constraint rectification: update policy based on the NPG of an unsatisfied constraint when constraints are not all approximately satisfied.

Algorithm 2 summarizes this three-step constrained multi-objective policy improvement framework and Algorithm 3 provides a concrete realization with our novel conflict-averse NPG method. Note, based on our theoretical guarantee on the time-average convergence, policy π_{out} can be uniformly chosen from $\mathcal{N}_0$, the detail proof is provided in section 4.8. To ease the presentation and better illustrate the main idea, we will focus on the tabular MDP setting in this section. The extension to the more practical setting of deep RL will be discussed in section 4.8.

Algorithm 2 CR-MOPO: Constraint-Rectified Multi-Objective Policy Optimization Framework

- 1: **Inputs:** initial parameter π_{w_0} , empty set $\mathcal{N}_0$.
 - 2: **for** $t = 0, \dots, T - 1$ **do**
 - 3: Policy evaluation under π_{w_t} for all objectives and constraints.
 - 4: **if** constraints are all satisfied **then**
 - 5: Add π_{w_t} into set $\mathcal{N}_0$.
 - 6: Compute the multi-objective policy update direction $\mathbf{d}$ and update policy using $\mathbf{d}$.
 - 7: **else**
 - 8: Choose any unsatisfied constraint i_t and update policy towards minimize $f_{i_t}(\pi_{w_t})$.
 - 9: **end if**
 - 10: **end for**
 - 11: **Outputs:** π_{out} uniformly chosen from $\mathcal{N}_0$.
-

4.4.1 Policy Evaluation of Safe Multi-Objective Reinforcement Learning

In this step, we aim to learn Q-functions that can effectively evaluate the preceding policy π_t . To achieve this, we train individual Q-functions for each objective and constraint. In principle, any Q-learning algorithm can be used, as long as the target Q-value is computed with respect to π_t .

Temporal difference (TD) learning. In TD learning, each iteration takes the form of

$$Q_{i,k+1}^{\pi_w}(s, a) = Q_{i,k}^{\pi_w} + \ell_k \left[r_i(s, a) + \gamma Q_{i,k}^{\pi_w}(s', a') - Q_{i,k}^{\pi_w}(s, a) \right], \quad (4.3)$$

where $s \sim \mu_{\pi_w}$, $a \sim \pi_w(s)$, $s' \sim P(\cdot | s, a)$, $a' \sim \pi_w(s')$, and ℓ_k is the learning rate. It has been shown in [28, 61] that the iteration in equation 4.3 converges to the fixed point which is the state-action value $Q_i^{\pi_w}$. After performing K_{TD} iterations of equation 4.3, we let the estimation $\bar{Q}_i(s, a) = Q_{i,K_{\text{TD}}}^{\pi_w}(s, a)$.

Unbiased Q-estimation. To obtain an unbiased estimation of the state-action value [286], we can perform Monte-Carlo rollouts for a trajectory with the horizon $H \sim \text{Geom}(1 - \gamma^{1/2})$, where $\text{Geom}(x)$ denotes a geometric distribution with parameter x , and estimate the state-action value function along the trajectory $(s_0, a_0, \dots, s_H, a_H)$ as follows:

$$\bar{Q}_i(s_0, a_0) = r_i(s_0, a_0) + \sum_{h=1}^H \gamma^{h/2} r_i(s_h, a_h). \quad (4.4)$$

4.4.2 Policy Improvement for Multi-Objectives

4.4.2.1 Conflict-Averse Natural Policy Gradient

The policy gradient [235] of the value function $f_i(\pi_w)$ has been derived as $\nabla f_i(\pi_w) = \mathbb{E}[Q_i^{\pi_w}(s, a) \phi_w(s, a)]$, where $\phi_w(s, a) := \nabla_w \log \pi_w(a | s)$ is the score function. However, the standard policy gradient does not effectively reflect the statistical manifold (the family of probability distributions that represents the policy function) that the policy operates on. To prevent the policy itself from changing too much during an update, we need to consider how sensitive the policy is to parameter changes.

Thus, in the multi-objectives policy optimization, we aim to choose an update direction $\mathbf{d}$ to increase every individual value function while imposing the constraint on the allowed changes of an update in terms of the KL divergence of the policy. To do so, we consider the following constrained optimization problem:

$$\max_{\mathbf{d}: D_{\text{KL}}(\pi_w | \pi_{w+\mathbf{d}}) \leq \epsilon_0} \min_{i \in [m]} \{ \xi_i (f_i(w + \mathbf{d}) - f_i(w)) \} \quad (4.5)$$

where ϵ_0 is the pre-defined threshold for allowed policy changes. By using the first-order Taylor approximation for the value improvement, the second-order Taylor approximation for the KL divergence constraint and the Lagrangian relaxation, the problem equation 4.5 can be rewritten as

$$\max_{\mathbf{d}} \min_{i \in [m]} \left\{ \xi_i \nabla f_i(w)^\top \mathbf{d} - \frac{\psi_1}{2} \mathbf{d}^\top \tilde{F}(w) \mathbf{d} \right\}, \quad (4.6)$$

where $\psi_1 > 0$ is a pre-specified hyper-parameter to control the allowed changes in policy space and $\tilde{F}(w)$ is the Fisher information matrix defined as $\tilde{F}(w) = \nabla_{w'}^2 D_{\text{KL}}(\pi_w | \pi_{w'})|_{w'=w} = \mathbb{E}_{\nu_{\pi_w}} [\phi_w(s, a) \phi_w(s, a)^\top]$. For a single objective f_i , the solution of equation 4.6 leads to the well-known NPG update [118] which is defined as $\tilde{F}(w)^\dagger \nabla f_i(\pi_w)$. Note that TRPO [216] can be viewed as the NPG approach with adaptive stepsize.

With the above problem formulation, we aim to find an update direction that minimizes the gradient conflicts. Furthermore, inspired by the recent advances in gradient manipulation method [147] which looks for the best update direction within a local ball centered at the

weighted averaged gradient, we also constraint search region for the common direction as a circle around the weighted average policy gradient $\mathbf{v}_0 = \sum_{i=1}^m \xi_i \nabla f_i(w)$. This yields **Conflict-Averse Natural Policy Gradient (CA-NPG)** which determines the update direction $\mathbf{d}$ by solving the following optimization problem

$$\max_{\mathbf{d}} \min_{i \in [m]} \left\{ \xi_i \nabla f_i(w)^\top \mathbf{d} - \frac{\psi_1}{2} \mathbf{d}^\top \tilde{F}(w) \mathbf{d} - \frac{\psi_2}{2} \|\mathbf{d} - \mathbf{v}_0\|^2 \right\}, \quad (4.7)$$

where $\psi_2 > 0$ is a pre-specified hyper-parameter that controls the deviation from the weighted average policy gradient $\mathbf{v}_0$. Furthermore, notice that

$$\min_i \xi_i \nabla f_i(w)^\top \mathbf{d} = \min_{\boldsymbol{\theta} \in S_m} \sum_{i \in [m]} \theta_i \xi_i \nabla f_i(w)^\top \mathbf{d},$$

where $\boldsymbol{\theta} = (\theta_1, \dots, \theta_m)$ and $S_m = \{\boldsymbol{\theta} : \sum_{i=1}^m \theta_i = 1, \theta_i \geq 0\}$. Denote $\nabla f_{\boldsymbol{\theta}}(w) = \sum_{i \in [m]} \theta_i \xi_i \nabla f_i(w)$. The objective in equation 4.7 can be written as

$$\max_{\boldsymbol{\theta} \in S_m} \min_{\mathbf{d}} \left\{ \nabla f_{\boldsymbol{\theta}}(w)^\top \mathbf{d} - \frac{\psi_1}{2} \mathbf{d}^\top \tilde{F}(w) \mathbf{d} - \frac{\psi_2}{2} \|\mathbf{d} - \mathbf{v}_0\|^2 \right\}.$$

Since the above objective is concave with respect to $\mathbf{d}$ and linear with respect to $\boldsymbol{\theta}$, by switching the min and max, we reach the dual form without changing the solution:

$$\min_{\boldsymbol{\theta} \in S_m} \max_{\mathbf{d}} \left\{ \nabla f_{\boldsymbol{\theta}}(w)^\top \mathbf{d} - \frac{\psi_1}{2} \mathbf{d}^\top \tilde{F}(w) \mathbf{d} - \frac{\psi_2}{2} \|\mathbf{d} - \mathbf{v}_0\|^2 \right\}.$$

After a few steps of calculus, we derive the following optimization problem with respect to the variable $\boldsymbol{\theta}$:

$$\begin{aligned} \boldsymbol{\theta}^* = \arg \min_{\boldsymbol{\theta} \in S_m} & \nabla f_{\boldsymbol{\theta}}^\top (\psi_1 \tilde{F} + \psi_2 I)^{-1} (\nabla f_{\boldsymbol{\theta}} + \psi_2 \mathbf{v}_0) \\ & - \frac{\psi_1}{2} (\nabla f_{\boldsymbol{\theta}} + \psi_2 \mathbf{v}_0)^\top (\psi_1 \tilde{F} + \psi_2 I)^{-1} \\ & \tilde{F} (\psi_1 \tilde{F} + \psi_2 I)^{-1} (\nabla f_{\boldsymbol{\theta}} + \psi_2 \mathbf{v}_0) \\ & - \frac{\psi_2}{2} \left\| (\psi_1 \tilde{F} + \psi_2 I)^{-1} (\nabla f_{\boldsymbol{\theta}} + \psi_2 \mathbf{v}_0) \right\|^2, \end{aligned}$$

and the optimal update direction is given by

$$\mathbf{d}^* := \boldsymbol{\lambda}^\top \nabla \mathbf{F}^\psi = \sum_{i \in [m]} \lambda_i (\psi_1 \tilde{F} + \psi_2 I)^{-1} \nabla f_i, \quad (4.8)$$

where $\lambda_i = \xi_i (\theta_i^* + \psi_2)$ and $\nabla \mathbf{F}_i^\psi = (\psi_1 \tilde{F} + \psi_2 I)^{-1} \nabla f_i$. To simplify the notation, we omit the superscript ψ in $\nabla \mathbf{F}^\psi$ for the subsequent sections.

4.4.2.2 Correlation-Reduction for Stochastic Gradient Manipulation

In practice, we only obtain noisy policy gradient feedback $\widehat{\nabla \mathbf{F}}(w_t)$, where the stochastic noise is due to the finite sampled trajectories for the estimation of $Q_i^{\pi_{w_t}}$. It has been shown in [292] that the gradient manipulation methods may fail to converge to a Pareto optimal solution under the stochastic setting. This convergence gap is mainly caused by the strong correlation between the weights $\boldsymbol{\lambda}_t$ and the stochastic gradients $\widehat{\nabla \mathbf{F}}(w_t)$ which yields a biased composite gradient. To address this issue in CA-NPG, we consider two conditions. The first is that the NPG estimator variance asymptotically converges to 0. For example, this can be achieved by estimating $Q_i^{\pi_{w_t}}$ using TD learning in equation 4.3 with sufficiently large K_{TD} . The second is to reduce the variances of $\boldsymbol{\lambda}_\tau$ by adopting a momentum mechanism [292] with coefficient α_τ on the update of composite weights

$$\hat{\boldsymbol{\lambda}}_\tau = \alpha_\tau \hat{\boldsymbol{\lambda}}_{\tau-1} + (1 - \alpha_\tau) \boldsymbol{\lambda}_\tau, \quad (4.9)$$

where $\boldsymbol{\lambda}_\tau$ is computed by CA-NPG algorithms.

4.4.3 Constraint Rectification

We then check whether there exists a hard constraint $i \in \{m+1, \dots, m+p\}$ such that the (approximated) constraint function violates the condition. If so, we take one-step update of the policy using NPG towards minimizing the corresponding constraint function $f_i(\pi_{w_t})$ to enforce the constraint:

$$w_{t+1} = w_t - \eta \tilde{F}(w)^{\dagger} \nabla f_i(\pi_w).$$

If multiple constraints are violated, we can choose to minimize any one of them. Otherwise, we take one update of the policy towards maximizing the multi-objectives.

4.4.4 Comparison with Learning Preferences and Policies in Parallel

Compared with SOTA safe multi-objective RL method, LP3 [107], our new framework is different in both multi-objective optimization and hard constraint satisfaction. Firstly, LP3 chooses MO-MPO [2] as the multi-objective optimizer which encodes the objective preferences in a scale-invariant way through the allowed KL divergence for the updated policy using each objective. On the other hand, our multi-objective optimization method is based on linear scalarization coupled with novel NPG manipulation which encodes the preference in a more straightforward way and is tailored to RL to address the conflicting gradients and dominating gradients. Secondly, LP3 can be regarded as a primal-dual approach where the additional dual variables are introduced as the adaptive weights for the constraints. This relaxes hard constraints in safe multi-objective RL problems to new objectives where the associated weights are adjusted based on the constraint violation conditions. On the other hand, our primal-based method does not suffer from extra hyperparameter tuning and dual update and can be implemented as easily as unconstrained policy optimization algorithms.

Algorithm 3 CR-MOPO with CA-NPG as multi-objective optimizer.

- 1: **Inputs:** initial parameter w_0 , empty set $\mathcal{N}_0$, $\tau = 0$.
 - 2: **for** $t = 0, \dots, T-1$ **do**
 - 3: Policy evaluation under $\pi_{w_t} : \bar{Q}_i^t(s, a) \approx Q_i^{\pi_{w_t}}(s, a)$ for all $i = 1, \dots, m+p$.
 - 4: Collect pairs $(s^j, a^j) \in \mathcal{B}_t \sim \rho \cdot \pi_{w_t}$, compute constrain estimation $\bar{J}_{i, \mathcal{B}_t} = \sum_{j \in \mathcal{B}_t} \frac{1}{|\mathcal{B}_t|} \bar{Q}_i^t(s^j, a^j)$ for all $i = 1, \dots, m+p$, where j is the index for the sampled pairs in $\mathcal{B}_t$.
 - 5: **if** $\bar{J}_{i, \mathcal{B}_t} \leq c_i + \beta$ for all $i = m+1, \dots, m+p$ **then**
 - 6: $\tau \leftarrow \tau + 1$; add w_t into set $\mathcal{N}_0$.
 - 7: Compute the weights λ_τ using equation 4.8 and reduce the correlation by equation 4.9.
 - 8: Compute the multi-objective policy gradient $\mathbf{d}_\tau = \hat{\lambda}_\tau^\top \widehat{\nabla F}(w_t)$.
 - 9: Take one-step policy update: $w_{t+1} = w_t + \eta \mathbf{d}_\tau$.
 - 10: **else**
 - 11: Choose any $i_t \in \{m+1, \dots, m+p\}$ such that $\bar{J}_{i_t, \mathcal{B}_t} > c_{i_t} + \beta$.
 - 12: Take one-step policy update towards minimize $J_{i_t}(w_t)$: $w_{t+1} \leftarrow w_t - \eta \tilde{F}(w_t)^{\dagger} \nabla f_{i_t}(w_t)$.
 - 13: **end if**
 - 14: **end for**
 - 15: **Outputs:** w_{out} uniformly chosen from $\mathcal{N}_0$.
-

4.5 Theoretical Analysis of Safe Multi-Objective Reinforcement Learning

In this section, we establish the convergence and the constraint violation guarantee for CR-MOPO in the tabular settings under the softmax parameterization and CA-NPG. In the tabular setting, we consider the softmax parameterization. For any $w \in \mathbb{R}^{|\mathcal{S}| \times |\mathcal{A}|}$, the corresponding softmax policy π_w is defined as $\pi_w(a | s) := \frac{\exp(w(s,a))}{\sum_{a' \in \mathcal{A}} \exp(w(s,a'))}$, $\forall (s, a) \in \mathcal{S} \times \mathcal{A}$. Clearly, the policy class defined above is complete, as any stochastic policy in the tabular setting can be represented in this class. Since the adaptive weights λ_t of CA-NPG may not be constrained in the probability simplex. Hence, we consider the following mild assumption on the boundedness of λ_t .

Assumption 1. *For the CA-NPG mechanism, there exists finite constants $B_1 > 0$ and $B_2 > 0$ such that $0 \leq \lambda_t^i \leq B_1$, $\sum_{i=1}^m \lambda_t^i \geq B_2$ for all $t = 1, \dots, T$, $i = 1, \dots, m$.*

For multi-objective optimization, if there exists $\lambda^* \in S_m$ such that $w^* = \arg \min_w \lambda^{*\top} F(\pi_w)$, then w^* is (weak) Pareto optimal [Theorem 5.13 and Lemma 5.14 in [115]]. Thus, we use $\min_{\lambda^* \in S_m} (\lambda^{*\top} F(\pi^*) - \lambda^{*\top} F(\pi_{w_{out}}))$ to measure the convergence to a Pareto optimal policy where minimization operator of λ^* is from the existence condition. The following theorem characterizes the convergence rate of Algorithm 3 in terms of the Pareto optimal policy convergence and hard constraint violations. The proof can be found in the appendix 4.8.

Theorem 3. *Consider Algorithm 3 in the tabular setting with softmax policy parameterization and any policy initialization $w_0 \in \mathcal{R}^{|\mathcal{S}| \times |\mathcal{A}|}$. Let the tolerance be $\beta = \mathcal{O}\left(\frac{mB_1 \sqrt{|\mathcal{S}| |\mathcal{A}|}}{(1-\gamma)^2 \sqrt{T}}\right)$ and the learning rate for the CA-NPG and NPG be $\eta = \mathcal{O}\left(\frac{(1-\gamma)^2}{mB_1 \sqrt{|\mathcal{S}| |\mathcal{A}| T}}\right)$. Depending on the choice of the state-action value estimator, the following holds.*

- If TD-learning in equation 4.3 is used for policy evaluation with $K_{TD} = \tilde{\mathcal{O}}\left(\left(\frac{T}{(1-\gamma)^2 |\mathcal{S}| |\mathcal{A}|}\right)^{\frac{1}{\sigma}}\right)$, $\ell_k = \mathcal{O}(\frac{1}{k^\sigma})$ and $\alpha_\tau = 0$ for $0 < \sigma < 1$, then with probability $1 - \delta$, we have

$$\begin{aligned} \mathbb{E} \left[\min_{\lambda^* \in S_m} (\lambda^{*\top} F(\pi^*) - \lambda^{*\top} F(\pi_{w_{out}})) \right] &\leq \frac{\beta}{B_2}, \\ \mathbb{E} [f_i(\pi_{w_{out}})] - c_i &\leq \beta, \end{aligned}$$

for all $i = \{m+1, \dots, m+p\}$, where the expectation is taken only with respect to selecting w_{out} from $\mathcal{N}_0$.

- If unbiased Q-estimation in equation 4.4 is used for policy evaluation with $\alpha_\tau \geq 1 - \frac{1-\gamma}{m\tau \sqrt{|\mathcal{S}| |\mathcal{A}|}}$, we have

$$\begin{aligned} \mathbb{E} \left[\min_{\lambda^* \in S_m} (\lambda^{*\top} F(\pi^*) - \lambda^{*\top} F(\pi_{w_{out}})) \right] &\leq \frac{\beta}{B_2}, \\ \mathbb{E} [f_i(\pi_{w_{out}})] - c_i &\leq \beta, \end{aligned}$$

for all $i = \{m+1, \dots, m+p\}$, where the expectation is taken with respect to selecting w_{out} from $\mathcal{N}_0$ and the randomness of $Q_{\pi_{w_t}}^i$ estimation.

As shown in Theorem 3, our method is guaranteed to find a safe Pareto optimal policy under some mild conditions while there is no convergence guarantee for LP3 [107]. Furthermore, results for unbiased Q-estimation imply that the correlation reduction mechanism could help the convergence even if we do not have an asymptotically increasing trajectory for policy evaluation, such as $K_{TD} = \tilde{\mathcal{O}}(T^{1/\sigma})$ in TD-learning.

4.6 Experiments of Safe Multi-Objective Reinforcement Learning

Environment Settings. We have developed a benchmark, referred to as *Safe Multi-Objective MuJoCo*, to evaluate our algorithms within the MuJoCo framework [39, 244]. A comprehensive description of this benchmark is provided in Section 6.7.1, where we introduce environments such as Safe Multi-Objective HalfCheetah, Safe Multi-Objective Hopper, Safe Multi-Objective Humanoid, Safe Multi-Objective Swimmer, Safe Multi-Objective Walker, and Safe Multi-Objective Pusher to assess the effectiveness of our proposed methods. For a detailed description of the environment settings, we kindly refer the reader to Section 6.7.1.

CR-MOPO exhibits strong performance in challenging safe multi-objective environments. As illustrated in Figure 4.1, a series of experiments were conducted across various challenging tasks. The cost limits for each step were set as follows: HalfCheetah-v4 at 0.1, Humanoid-v4 at 0.9, Walker-v4 at 0.03, Pusher-v4 at 0.49, and Hopper-v4-v2 at 0.045. The optimization of safety violations was performed after 40 Epochs for all tasks, with the exception of the Humanoid-dm task, for which the optimization was carried out after 5 Epochs. The experimental results demonstrate that our method is capable of ensuring monotonic improvement in each task’s reward while maintaining safety across all challenging tasks. For further experimental details, refer to Appendix 6.7.1.

Exploiting constraints’ dual nature elevates both safety and performance. In this study, we investigate *the implications of incorporating a safety constraint both as a standalone constraint and an ancillary objective within the multi-objective framework*, referred to as CR-MOPO-Soft (CR-MOPO-S). Within this framework, the constraint is seamlessly integrated by allocating a specified weight to performance, for instance, a weight of 1.0. Conversely, CR-MOPO does not share any functions for its objectives and constraints.

We postulate that treating the constraint function as an objective can effectively buffer the feasible set’s boundary, facilitating navigation towards a “deep safe” set. This, in turn, ensures uninterrupted progress in performance. Such a mechanism becomes indispensable in systems heavily constrained, where proximity to the safety boundary might induce constraint breaches and erratic behaviors, as observed in prevalent safe learning methodologies like CRPO [266], CPO [5], and PCPO [271].

To validate our hypothesis, we amplify the scope of our experiments and scenarios, centering on a simple Constrained Markov Decision Process (CMDP) setup. Within this context, the emphasis lies on reward optimization for a singular objective, alongside incorporating safety constraints. We position CR-MOPO-S in contrast to CR-MOPO, which omits the constraint from its objective, and set it against the benchmark of the safe RL paradigm, CRPO [266]. It’s worth noting that CRPO exemplifies a safe RL algorithm that has consistently showcased its efficacy in harmonizing reward and safety violation, surpassing other safe RL algorithms like PDO [208]. Given CRPO’s design for single objective safe RL, we aggregate multiple objectives into a singular objective to align with their algorithm. Comprehensive implementation details are in Appendix 6.7.2.

As illustrated in Figure 4.2, our introduced algorithms, CR-MOPO and CR-MOPO-S, surpass CRPO in both reward and safety metrics. Notably, CR-MOPO-S showcases superior constraint adherence and performance optimization compared to its sibling, CR-MOPO. An intuitive way to understand this is by likening it to running on a road. If one veers too close to its edge, any misstep would mean needing to reposition, potentially compromising speed. However, by choosing to run centrally, both safety and optimal speed can be achieved simultaneously. From an algorithmic perspective, while constraints present a binary outcome (satisfied or not), objectives pursue a continuous improvement trajectory. Softening this binary facet of constraints allows us to adeptly navigate challenging scenarios, circumventing the frequent toggling often seen at the feasible set’s perimeter. *This nuanced handling is a hallmark of our*

constrained multiobjective RL framework.

Benefit of boundary-aware policy learning (in comparison to LP3 [107]). In this section, we assess our algorithm against LP3 [107] using identical environmental settings as detailed in [107]. LP3, a SOTA safe multi-objective RL benchmark, aligns policy optimization with constraints by determining preferences over objectives. It has been successfully implemented in demanding tasks such as Humanoid-dm and Walker-dm from the DeepMind Control Suite [239].

As evidenced in Figure 4.3, our algorithm consistently improves over LP3 [107] while preserving safety. Specifically, in the Walker-dm task (refer to Figure 4.3 (a) and (b)), with a cost limit of 1.5, our technique achieves move and height rewards surpassing 800 and 600 respectively, while LP3 manages roughly 250 and 800 in the same measures. Regarding the Humanoid-dm task, while LP3 scores around 400 and 300 for the move left and move forward rewards with a cost limit of 1.5, our approach consistently reaches at least 400 for move left and approximately 600 for move forward rewards. These empirical findings clearly underscore the edge of our algorithm over LP3 [107]. Our method’s edge stems from its unique capability to dynamically adjust the policy learning strategy based on proximity to constraint boundaries. While positioned within the safety set, our algorithm *eases* constraint satisfaction, potentially sidestepping conflicting gradient updates that can arise closer to the boundaries. This boundary-aware learning contrasts with LP3’s approach, which, by converting constraints into objectives through preference delineation, fails to tap into the full breadth of the safety set. While our strategy may sometimes favor policies near the safety boundary, judiciously selecting more stringent constraint thresholds can readily mitigate such tendencies.

4.7 Summary of Safe Multi-Objective Reinforcement Learning

In this study, we aim to strike a balance between the performance of individual objectives while maintaining safety in a multi-objective RL context. To this end, a primal-based safe multi-objective RL framework is proposed, which resolves multiple conflict gradients through gradient manipulation and employs constraint rectification to identify safety policies during multi-objective optimization. Moreover, the analysis of convergence and safety violations are provided. In conclusion, we deploy our practical algorithms on several challenging, safe multi-objective RL environments and compare our method with the SOTA safe RL baseline and safe multi-objective RL algorithms. The experiment results indicate that our method can perform better than SOTA-safe RL baselines and SOTA-safe multi-objective RL algorithms regarding the balance between each objective performance and safety violation. In future, we plan to deploy our algorithm in real-world scenarios and try to leverage the foundation models [270] with our method to address safe multi-objective RL robustness problems.

4.8 Proofs of Safe Multi-Objective Reinforcement Learning

We prove the results of Theorem 3 in two parts. We give the TD-Learning results in Theorem 4 in section 4.8.1, and the unbiased q-estimation estimator results in Theorem 5 in section 4.8.2.

Lemma 1 (Multi-objective NPG). *Given the preference vector λ and considering the multi-objective NPG update $w_{t+1} = w_t - \eta \tilde{F}(w_t)^\dagger \sum_{i=1}^m \lambda^i \nabla f_i(\pi_{w_t})$, where $\tilde{F}(w) = \mathbb{E}_{\nu_{\pi_w}} [\phi_w(s, a) \phi_w(s, a)^\top]$*

in the tabular setting, the multi-objective NPG update also takes the form:

$$w_{t+1} = w_t + \frac{\eta}{1-\gamma} \sum_{i=1}^m \lambda^i Q_i^{\pi_{w_t}},$$

$$\pi_{w_{t+1}}(a | s) = \pi_{w_t}(a | s) \frac{\exp\left(\eta \sum_{i=1}^m \lambda_i Q_i^{\pi_{w_t}} / (1-\gamma)\right)}{Z_t(s)},$$

where

$$Z_t(s) = \sum_{a \in \mathcal{A}} \pi_{w_t}(a | s) \exp\left(\frac{\eta \sum_{i=1}^m \lambda_i Q_i^{\pi_{w_t}}(s, a)}{1-\gamma}\right).$$

Proof. From Lemma 5.1 in [6], we know

$$\tilde{F}(w_t)^\dagger \nabla f_i(\pi_{w_t}) = \frac{A_i^{\pi_{w_t}}}{1-\gamma} + v,$$

where $v \in \mathbb{R}^{|\mathcal{S}||\mathcal{A}|}$ and $v_{s,a} = c_s$ for some $c_s \in \mathbb{R}$ for each state s and action a . This yields the updates

$$w_{t+1} = w_t + \frac{\eta}{1-\gamma} \sum_{i=1}^m \lambda^i A_i^{\pi_{w_t}} + \eta v \sum_{i=1}^m \lambda^i$$

and

$$\pi_{w_{t+1}}(a | s) = \pi_{w_t}(a | s) \frac{\exp\left(\eta \sum_{i=1}^m \lambda_i A_i^{\pi_{w_t}} / (1-\gamma) + \eta c_s \sum_{i=1}^m \lambda_i\right)}{Z_t(s)}.$$

Owing to the normalization factor $Z_t(s)$, the state-dependent offset c_s cancels in the updates for π , so that resulting policy is invariant to the specific choice of c_s . Hence, we pick $c_s \equiv 0$, which yields the updates

$$w_{t+1} = w_t + \frac{\eta}{1-\gamma} \sum_{i=1}^m \lambda^i A_i^{\pi_{w_t}}$$

and

$$\pi_{w_{t+1}}(a | s) = \pi_{w_t}(a | s) \frac{\exp\left(\eta \sum_{i=1}^m \lambda^i A_i^{\pi_{w_t}} / (1-\gamma)\right)}{Z_t(s)}.$$

Finally, the advantage function $A_i^{\pi_w}(s, a) = Q_i^{\pi_w}(s, a) - V_i^{\pi_w}(s, a)$ can be replaced by the Q-function $Q_i^{\pi_w}(s, a)$ due to the normalization factor $Z_t(s)$, which yields the the statement of the lemma. $\square$

Lemma 2 (Performance improvement bound for approximated multi-objective NPG). *For the iterates π_{w_t} generated by the approximated multi-objective NPG updates in the tabular setting, we have the following holds*

$$\begin{aligned} & \sum_{i=1}^m \lambda^i (f_i(w_{t+1}) - f_i(w_t)) \\ & \geq \frac{1-\gamma}{\eta} \mathbb{E}_{s \sim \rho} \left(\log Z_t(s) - \frac{\eta}{1-\gamma} \sum_{i=1}^m \lambda^i V_i^{\pi_{w_t}}(s) + \frac{\eta}{1-\gamma} \sum_{a \in \mathcal{A}} \pi_{w_t}(a | s) \sum_{i=1}^m \lambda^i (Q_i^{\pi_{w_t}}(s, a) - \bar{Q}_i^t(s, a)) \right) \\ & \quad - \frac{1}{1-\gamma} \mathbb{E}_{s \sim \nu_\rho} \sum_{a \in \mathcal{A}} \pi_{w_t}(a | s) \sum_{i=1}^m \lambda^i (Q_i^{\pi_{w_t}}(s, a) - \bar{Q}_i^t(s, a)) \\ & \quad + \frac{1}{1-\gamma} \mathbb{E}_{s \sim \nu_\rho} \sum_{a \in \mathcal{A}} \pi_{w_{t+1}}(a | s) \sum_{i=1}^m \lambda^i (Q_i^{\pi_{w_t}}(s, a) - \bar{Q}_i^t(s, a)). \end{aligned}$$

Proof. We first provide the following lower bound.

$$\begin{aligned}
& \log Z_t(s) - \frac{\eta}{1-\gamma} \sum_{i=1}^m \lambda^i V_i^{\pi_{w_t}}(s) \\
&= \log \sum_{a \in \mathcal{A}} \pi_{w_t}(a | s) \exp \left(\frac{\eta \sum_{i=1}^m \lambda^i \bar{Q}_i^t(s, a)}{1-\gamma} \right) - \frac{\eta}{1-\gamma} \sum_{i=1}^m \lambda^i V_i^{\pi_{w_t}}(s) \\
&\geq \sum_{a \in \mathcal{A}} \pi_{w_t}(a | s) \log \exp \left(\frac{\eta \sum_{i=1}^m \lambda^i \bar{Q}_i^t(s, a)}{1-\gamma} \right) - \frac{\eta}{1-\gamma} \sum_{i=1}^m \lambda^i V_i^{\pi_{w_t}}(s) \\
&= \frac{\eta}{1-\gamma} \sum_{a \in \mathcal{A}} \pi_{w_t}(a | s) \sum_{i=1}^m \lambda^i \left(\bar{Q}_i^t(s, a) - Q_i^{\pi_{w_t}}(s, a) \right) \\
&\quad + \frac{\eta}{1-\gamma} \sum_{a \in \mathcal{A}} \pi_{w_t}(a | s) \sum_{i=1}^m \lambda^i Q_i^{\pi_{w_t}}(s, a) - \frac{\eta}{1-\gamma} \sum_{i=1}^m \lambda^i V_i^{\pi_{w_t}}(s) \\
&= \frac{\eta}{1-\gamma} \sum_{a \in \mathcal{A}} \pi_{w_t}(a | s) \sum_{i=1}^m \lambda^i \left(\bar{Q}_i^t(s, a) - Q_i^{\pi_{w_t}}(s, a) \right).
\end{aligned}$$

Thus, we conclude that

$$\log Z_t(s) - \frac{\eta}{1-\gamma} \sum_{i=1}^m \lambda^i V_i^{\pi_{w_t}}(s) + \frac{\eta}{1-\gamma} \sum_{a \in \mathcal{A}} \pi_{w_t}(a | s) \sum_{i=1}^m \lambda^i \left(Q_i^{\pi_{w_t}}(s, a) - \bar{Q}_i^t(s, a) \right) \geq 0. \quad (4.10)$$

We then proceed to prove this lemma. The performance difference lemma [118] implies:

$$\begin{aligned}
& \sum_{i=1}^m \lambda^i (f_i(w_{t+1}) - f_i(w_t)) \\
&= \frac{1}{1-\gamma} \mathbb{E}_{s \sim \nu_\rho} \sum_{a \in \mathcal{A}} \pi_{w_{t+1}}(a | s) \sum_{i=1}^m \lambda^i A_i^{\pi_{w_t}}(s, a) \\
&= \frac{1}{1-\gamma} \mathbb{E}_{s \sim \nu_\rho} \sum_{a \in \mathcal{A}} \pi_{w_{t+1}}(a | s) \sum_{i=1}^m \lambda^i Q_i^{\pi_{w_t}}(s, a) - \frac{1}{1-\gamma} \mathbb{E}_{s \sim \nu_\rho} \sum_{i=1}^m \lambda^i V_i^{\pi_{w_t}}(s) \\
&= \frac{1}{1-\gamma} \mathbb{E}_{s \sim \nu_\rho} \sum_{a \in \mathcal{A}} \pi_{w_{t+1}}(a | s) \sum_{i=1}^m \lambda^i \bar{Q}_i^t(s, a) \\
&\quad + \frac{1}{1-\gamma} \mathbb{E}_{s \sim \nu_\rho} \sum_{a \in \mathcal{A}} \pi_{w_{t+1}}(a | s) \sum_{i=1}^m \lambda^i (Q_i^{\pi_{w_t}}(s, a) - \bar{Q}_i^t(s, a)) - \frac{1}{1-\gamma} \mathbb{E}_{s \sim \nu_\rho} \sum_{i=1}^m \lambda^i V_i^{\pi_{w_t}}(s) \\
&\stackrel{(i)}{=} \frac{1}{\eta} \mathbb{E}_{s \sim \nu_\rho} \sum_{a \in \mathcal{A}} \pi_{w_{t+1}}(a | s) \log \left(\frac{\pi_{w_{t+1}}(a | s) Z_t(s)}{\pi_{w_t}(a | s)} \right) \\
&\quad + \frac{1}{1-\gamma} \mathbb{E}_{s \sim \nu_\rho} \sum_{a \in \mathcal{A}} \pi_{w_{t+1}}(a | s) \sum_{i=1}^m \lambda^i (Q_i^{\pi_{w_t}}(s, a) - \bar{Q}_i^t(s, a)) - \frac{1}{1-\gamma} \mathbb{E}_{s \sim \nu_\rho} \sum_{i=1}^m \lambda^i V_i^{\pi_{w_t}}(s) \\
&= \frac{1}{\eta} \mathbb{E}_{s \sim \nu_\rho} D_{\text{KL}}(\pi_{w_{t+1}} || \pi_{w_t}) + \frac{1}{\eta} \mathbb{E}_{s \sim \nu_\rho} \log Z_t(s) \\
&\quad + \frac{1}{1-\gamma} \mathbb{E}_{s \sim \nu_\rho} \sum_{a \in \mathcal{A}} \pi_{w_{t+1}}(a | s) \sum_{i=1}^m \lambda^i (Q_i^{\pi_{w_t}}(s, a) - \bar{Q}_i^t(s, a)) - \frac{1}{1-\gamma} \mathbb{E}_{s \sim \nu_\rho} \sum_{i=1}^m \lambda^i V_i^{\pi_{w_t}}(s) \\
&\geq \frac{1}{\eta} \mathbb{E}_{s \sim \nu_\rho} \left(\log Z_t(s) - \frac{\eta}{1-\gamma} \sum_{i=1}^m \lambda^i V_i^{\pi_{w_t}}(s) + \frac{\eta}{1-\gamma} \sum_{a \in \mathcal{A}} \pi_{w_t}(a | s) \sum_{i=1}^m \lambda^i (Q_i^{\pi_{w_t}}(s, a) - \bar{Q}_i^t(s, a)) \right) \\
&\quad - \frac{1}{1-\gamma} \mathbb{E}_{s \sim \nu_\rho} \sum_{a \in \mathcal{A}} \pi_{w_t}(a | s) \sum_{i=1}^m \lambda^i (Q_i^{\pi_{w_t}}(s, a) - \bar{Q}_i^t(s, a)) \\
&\quad + \frac{1}{1-\gamma} \mathbb{E}_{s \sim \nu_\rho} \sum_{a \in \mathcal{A}} \pi_{w_{t+1}}(a | s) \sum_{i=1}^m \lambda^i (Q_i^{\pi_{w_t}}(s, a) - \bar{Q}_i^t(s, a)) \\
&\stackrel{(ii)}{\geq} \frac{1-\gamma}{\eta} \mathbb{E}_{s \sim \nu_\rho} \left(\log Z_t(s) - \frac{\eta}{1-\gamma} \sum_{i=1}^m \lambda^i V_i^{\pi_{w_t}}(s) + \frac{\eta}{1-\gamma} \sum_{a \in \mathcal{A}} \pi_{w_t}(a | s) \sum_{i=1}^m \lambda^i (Q_i^{\pi_{w_t}}(s, a) - \bar{Q}_i^t(s, a)) \right) \\
&\quad - \frac{1}{1-\gamma} \mathbb{E}_{s \sim \nu_\rho} \sum_{a \in \mathcal{A}} \pi_{w_t}(a | s) \sum_{i=1}^m \lambda^i (Q_i^{\pi_{w_t}}(s, a) - \bar{Q}_i^t(s, a)) \\
&\quad + \frac{1}{1-\gamma} \mathbb{E}_{s \sim \nu_\rho} \sum_{a \in \mathcal{A}} \pi_{w_{t+1}}(a | s) \sum_{i=1}^m \lambda^i (Q_i^{\pi_{w_t}}(s, a) - \bar{Q}_i^t(s, a)),
\end{aligned}$$

where (i) follows from the update rule in Lemma 1 and (ii) follows from the facts that $\|\nu_\rho/\rho\| \geq 1-\gamma$ and equation 4.10. $\square$

Lemma 3 (Expected optimality gap for approximated multi-objective NPG). *Consider the*

approximated NPG updates in the tabular setting. We have

$$\begin{aligned}
\sum_{i=1}^m \lambda^i (f_i(\pi^*) - f_i(\pi_{w_t})) &\leq \frac{1}{\eta} \mathbb{E}_{s \sim \nu^*} (D_{\text{KL}}(\pi^* \| \pi_{w_t}) - D_{\text{KL}}(\pi^* \| \pi_{w_{t+1}})) + \frac{2\eta r_{\max}}{(1-\gamma)^2} \sum_{i=1}^m \lambda^i \left\| \sum_{i=1}^m \lambda^i \bar{Q}_i^t \right\|_2 \\
&\quad + \frac{1}{(1-\gamma)^2} \mathbb{E}_{s \sim \nu_p} \sum_{a \in \mathcal{A}} \pi_{w_t}(a | s) \sum_{i=1}^m \lambda^i (Q_i^{\pi_{w_t}}(s, a) - \bar{Q}_i^t(s, a)) \\
&\quad - \frac{1}{(1-\gamma)^2} \mathbb{E}_{s \sim \nu_p} \sum_{a \in \mathcal{A}} \pi_{w_{t+1}}(a | s) \sum_{i=1}^m \lambda^i (Q_i^{\pi_{w_t}}(s, a) - \bar{Q}_i^t(s, a)) \\
&\quad + \frac{1}{1-\gamma} \mathbb{E}_{s \sim \nu^*} \sum_{a \in \mathcal{A}} \pi^*(a | s) \sum_{i=1}^m \lambda^i (Q_i^{\pi_{w_t}}(s, a) - \bar{Q}_i^t(s, a)).
\end{aligned}$$

Proof.

$$\begin{aligned}
\sum_{i=1}^m \lambda^i (f_i(\pi^*) - f_i(\pi_{w_t})) &= \frac{1}{1-\gamma} \mathbb{E}_{s \sim \nu^*} \sum_{a \in \mathcal{A}} \pi^*(a | s) \sum_{i=1}^m \lambda^i A_i^{\pi_{w_t}}(s, a) \\
&= \frac{1}{1-\gamma} \mathbb{E}_{s \sim \nu^*} \sum_{a \in \mathcal{A}} \pi^*(a | s) \sum_{i=1}^m \lambda^i Q_i^{\pi_{w_t}}(s, a) - \frac{1}{1-\gamma} \mathbb{E}_{s \sim \nu^*} \sum_{i=1}^m \lambda^i V_i^{\pi_{w_t}}(s) \\
&= \frac{1}{1-\gamma} \mathbb{E}_{s \sim \nu^*} \sum_{a \in \mathcal{A}} \pi^*(a | s) \sum_{i=1}^m \lambda^i \bar{Q}_i^t(s, a) + \frac{1}{1-\gamma} \mathbb{E}_{s \sim \nu^*} \sum_{a \in \mathcal{A}} \pi^*(a | s) \sum_{i=1}^m \lambda^i (Q_i^{\pi_{w_t}}(s, a) - \bar{Q}_i^t(s, a)) \\
&\quad - \frac{1}{1-\gamma} \mathbb{E}_{s \sim \nu^*} \sum_{i=1}^m \lambda^i V_i^{\pi_{w_t}}(s) \\
&\stackrel{(i)}{=} \frac{1}{\eta} \mathbb{E}_{s \sim \nu^*} \sum_{a \in \mathcal{A}} \pi^*(a | s) \log \frac{\pi_{w_{t+1}}(a | s) Z_t(s)}{\pi_{w_t}(a | s)} + \frac{1}{1-\gamma} \mathbb{E}_{s \sim \nu^*} \sum_{a \in \mathcal{A}} \pi^*(a | s) \sum_{i=1}^m \lambda^i (Q_i^{\pi_{w_t}}(s, a) - \bar{Q}_i^t(s, a)) \\
&\quad - \frac{1}{1-\gamma} \mathbb{E}_{s \sim \nu^*} \sum_{i=1}^m \lambda^i V_i^{\pi_{w_t}}(s) \\
&= \frac{1}{\eta} \mathbb{E}_{s \sim \nu^*} (D_{\text{KL}}(\pi^* \| \pi_{w_t}) - D_{\text{KL}}(\pi^* \| \pi_{w_{t+1}})) + \frac{1}{1-\gamma} \mathbb{E}_{s \sim \nu^*} \sum_{a \in \mathcal{A}} \pi^*(a | s) \sum_{i=1}^m \lambda^i (Q_i^{\pi_{w_t}}(s, a) - \bar{Q}_i^t(s, a)) \\
&\quad + \frac{1}{\eta} \mathbb{E}_{s \sim \nu^*} \left(\log Z_t(s) - \frac{\eta}{1-\gamma} \sum_{i=1}^m \lambda^i V_i^{\pi_{w_t}}(s) \right) \\
&\leq \frac{1}{\eta} \mathbb{E}_{s \sim \nu^*} (D_{\text{KL}}(\pi^* \| \pi_{w_t}) - D_{\text{KL}}(\pi^* \| \pi_{w_{t+1}})) \\
&\quad + \frac{1}{\eta} \mathbb{E}_{s \sim \nu^*} \left(\log Z_t(s) - \frac{\eta}{1-\gamma} \sum_{i=1}^m \lambda^i V_i^{\pi_{w_t}}(s) + \frac{\eta}{1-\gamma} \sum_{a \in \mathcal{A}} \pi_{w_t}(a | s) \sum_{i=1}^m \lambda^i (Q_i^{\pi_{w_t}}(s, a) - \bar{Q}_i^t(s, a)) \right) \\
&\quad + \frac{1}{1-\gamma} \mathbb{E}_{s \sim \nu^*} \sum_{a \in \mathcal{A}} \pi^*(a | s) \sum_{i=1}^m \lambda^i (Q_i^{\pi_{w_t}}(s, a) - \bar{Q}_i^t(s, a))
\end{aligned} \tag{4.11}$$

$$\begin{aligned}
& \sum_{i=1}^m \lambda^i (f_i(\pi^*) - f_i(\pi_{w_t})) \\
& \stackrel{(ii)}{\leq} \frac{1}{\eta} \mathbb{E}_{s \sim \nu^*} (D_{\text{KL}}(\pi^* \| \pi_{w_t}) - D_{\text{KL}}(\pi^* \| \pi_{w_{t+1}})) + \frac{1}{1-\gamma} \sum_{i=1}^m \lambda^i (f_i^{\nu^*}(w_{t+1}) - f_i^{\nu^*}(w_t)) \\
& + \frac{1}{(1-\gamma)^2} \mathbb{E}_{s \sim \nu_\rho} \sum_{a \in \mathcal{A}} \pi_{w_t}(a | s) \sum_{i=1}^m \lambda^i (Q_i^{\pi_{w_t}}(s, a) - \bar{Q}_i^t(s, a)) \\
& - \frac{1}{(1-\gamma)^2} \mathbb{E}_{s \sim \nu_\rho} \sum_{a \in \mathcal{A}} \pi_{w_{t+1}}(a | s) \sum_{i=1}^m \lambda^i (Q_i^{\pi_{w_t}}(s, a) - \bar{Q}_i^t(s, a)) \\
& + \frac{1}{1-\gamma} \mathbb{E}_{s \sim \nu^*} \sum_{a \in \mathcal{A}} \pi^*(a | s) \sum_{i=1}^m \lambda^i (Q_i^{\pi_{w_t}}(s, a) - \bar{Q}_i^t(s, a)) \\
& \stackrel{(iii)}{\leq} \frac{1}{\eta} \mathbb{E}_{s \sim \nu^*} (D_{\text{KL}}(\pi^* \| \pi_{w_t}) - D_{\text{KL}}(\pi^* \| \pi_{w_{t+1}})) + \frac{2r_{\max}}{(1-\gamma)^2} \sum_{i=1}^m \lambda^i \|w_{t+1} - w_t\|_2 \\
& + \frac{1}{(1-\gamma)^2} \mathbb{E}_{s \sim \nu_\rho} \sum_{a \in \mathcal{A}} \pi_{w_t}(a | s) \sum_{i=1}^m \lambda^i (Q_i^{\pi_{w_t}}(s, a) - \bar{Q}_i^t(s, a)) \\
& - \frac{1}{(1-\gamma)^2} \mathbb{E}_{s \sim \nu_\rho} \sum_{a \in \mathcal{A}} \pi_{w_{t+1}}(a | s) \sum_{i=1}^m \lambda^i (Q_i^{\pi_{w_t}}(s, a) - \bar{Q}_i^t(s, a)) \\
& + \frac{1}{1-\gamma} \mathbb{E}_{s \sim \nu^*} \sum_{a \in \mathcal{A}} \pi^*(a | s) \sum_{i=1}^m \lambda^i (Q_i^{\pi_{w_t}}(s, a) - \bar{Q}_i^t(s, a)) \\
& \stackrel{(iii)}{\leq} \frac{1}{\eta} \mathbb{E}_{s \sim \nu^*} (D_{\text{KL}}(\pi^* \| \pi_{w_t}) - D_{\text{KL}}(\pi^* \| \pi_{w_{t+1}})) + \frac{2\eta r_{\max}}{(1-\gamma)^2} \sum_{i=1}^m \lambda^i \left\| \sum_{i=1}^m \lambda^i \bar{Q}_i^t \right\|_2 \\
& + \frac{1}{(1-\gamma)^2} \mathbb{E}_{s \sim \nu_\rho} \sum_{a \in \mathcal{A}} \pi_{w_t}(a | s) \sum_{i=1}^m \lambda^i (Q_i^{\pi_{w_t}}(s, a) - \bar{Q}_i^t(s, a)) \\
& - \frac{1}{(1-\gamma)^2} \mathbb{E}_{s \sim \nu_\rho} \sum_{a \in \mathcal{A}} \pi_{w_{t+1}}(a | s) \sum_{i=1}^m \lambda^i (Q_i^{\pi_{w_t}}(s, a) - \bar{Q}_i^t(s, a)) \\
& + \frac{1}{1-\gamma} \mathbb{E}_{s \sim \nu^*} \sum_{a \in \mathcal{A}} \pi^*(a | s) \sum_{i=1}^m \lambda^i (Q_i^{\pi_{w_t}}(s, a) - \bar{Q}_i^t(s, a)),
\end{aligned} \tag{4.12}$$

where (i) and (iii) follow from Lemma 1, (ii) follows from Lemma 2, and (iii) follows from the Lipschitz property of $f_i^{\nu^*}(w)$ such that $f_i^{\nu^*}(w_{t+1}) - f_i^{\nu^*}(w_t) \leq \frac{2c_{\max}}{1-\gamma} \|w_{t+1} - w_t\|_2$. $\square$

4.8.1 TD-Learning

Lemma 4 ([61]). *Consider the iteration given in equation 4.3 with arbitrary initialization Q_0^i . Assume that the stationary distribution μ_{π_w} is not degenerate for all $w \in \mathbb{R}^{|S| \times |\mathcal{A}|}$. Let stepsize $\ell_k = \Theta\left(\frac{1}{k^\sigma}\right)$ ($0 < \sigma < 1$). Then, with probability at least $1 - \delta$, we have*

$$\|Q_K^i - Q_{\pi_w}^i\|_2 = \mathcal{O}\left(\frac{\log(|S|^2 |\mathcal{A}|^2 K^2 / \delta)}{(1-\gamma) K^{\sigma/2}}\right).$$

Note that σ can be arbitrarily close to 1. Lemma 2 implies that we can obtain an approximation $\bar{Q}_t^i$ such that $\|\bar{Q}_t^i - Q_{\pi_w}^i\|_2 = \tilde{\mathcal{O}}(1/\sqrt{K_{TD}})$ with high probability.

Lemma 5. *If*

$$\beta > \frac{2}{\eta T} \mathbb{E}_{s \sim \gamma^*} D_{\text{KL}}(\pi^* || \pi_{w_0}) + \frac{4\eta r_{\max}^2 |\mathcal{S}| |\mathcal{A}|}{(1-\gamma)^4 T} \left(\left[\sum_{t \in \mathcal{N}_0} \left(\sum_{i=1}^m \lambda_t^i \right)^2 \right] + T - |\mathcal{N}_0| \right) \quad (4.13)$$

$$+ \frac{6}{(1-\gamma)^2 T} \left(\sum_{t \in \mathcal{N}_0} \sum_{i=1}^m \lambda_t^i \|Q_{\pi_{w_t}}^i - \bar{Q}_t^i\|_2 + \sum_{i=m+1}^{m+p} \sum_{t \in \mathcal{N}_i} \|Q_{\pi_{w_t}}^i - \bar{Q}_t^i\|_2 \right), \quad (4.14)$$

then we have the following holds:

1. $\mathcal{N}_0 \neq \emptyset$,
2. One of the following two statements must hold,
 - (a) $|\mathcal{N}_0| \geq \frac{T}{2}$,
 - (b) $\sum_{t \in \mathcal{N}_0} \sum_{i=1}^m \lambda_t^i (f_i(\pi^*) - f_i(\pi_{w_t})) \leq 0$.

Proof. When $t \in \mathcal{N}_0$, from Lemma 3 we have

$$\begin{aligned} \sum_{i=1}^m \lambda_t^i (f_i(\pi^*) - f_i(\pi_{w_t})) &\leq \frac{1}{\eta} \mathbb{E}_{s \sim \gamma^*} (D_{\text{KL}}(\pi^* || \pi_{w_t}) - D_{\text{KL}}(\pi^* || \pi_{w_{t+1}})) \\ &\quad + \frac{2\eta c_{\max}^2 |\mathcal{S}| |\mathcal{A}|}{(1-\gamma)^4} \left(\sum_{i=1}^m \lambda_t^i \right)^2 + \frac{3}{(1-\gamma)^2} \sum_{i=1}^m \lambda_t^i \|Q_{\pi_{w_t}}^i - \bar{Q}_t^i\|_2. \end{aligned} \quad (4.15)$$

Similarly, when $t \in \mathcal{N}_i (i \neq 0)$ we have

$$\begin{aligned} f_i(\pi_{w_t}) - f_i(\pi^*) &\leq \frac{1}{\eta} \mathbb{E}_{s \sim \gamma^*} (D_{\text{KL}}(\pi^* || \pi_{w_t}) - D_{\text{KL}}(\pi^* || \pi_{w_{t+1}})) \\ &\quad + \frac{2\eta c_{\max}^2 |\mathcal{S}| |\mathcal{A}|}{(1-\gamma)^4} + \frac{3}{(1-\gamma)^2} \|Q_{\pi_{w_t}}^i - \bar{Q}_t^i\|_2. \end{aligned} \quad (4.16)$$

Taking the summation of equation 4.15 and equation 4.16 from $t = 0$ to $T - 1$ gives

$$\begin{aligned} &\sum_{t \in \mathcal{N}_0} \sum_{i=1}^m \lambda_t^i (f_i(\pi^*) - f_i(\pi_{w_t})) + \sum_{i=1}^p \sum_{t \in \mathcal{N}_i} (f_i(\pi_{w_t}) - f_i(\pi^*)) \\ &\leq \frac{1}{\eta} \mathbb{E}_{s \sim \gamma^*} D_{\text{KL}}(\pi^* || \pi_{w_0}) + \frac{2\eta c_{\max}^2 |\mathcal{S}| |\mathcal{A}|}{(1-\gamma)^4} \left(\sum_{t \in \mathcal{N}_0} \left(\sum_{i=1}^m \lambda_t^i \right)^2 + T - |\mathcal{N}_0| \right) \\ &\quad + \frac{3}{(1-\gamma)^2} \left(\sum_{t \in \mathcal{N}_0} \sum_{i=1}^m \lambda_t^i \|Q_{\pi_{w_t}}^i - \bar{Q}_t^i\|_2 + \sum_{i=m+1}^{m+p} \sum_{t \in \mathcal{N}_i} \|Q_{\pi_{w_t}}^i - \bar{Q}_t^i\|_2 \right). \end{aligned} \quad (4.17)$$

Note that when $t \in \mathcal{N}_i (i \neq 0)$, we have $\bar{f}_i(w_t) > c_i + \beta$ (line 9 in Algorithm 1), which implies that

$$\begin{aligned} f_i(\pi_{w_t}) - f_i(\pi^*) &\geq \bar{f}_i(w_t) - f_i(\pi^*) - |\bar{f}_i(w_t) - f_i(\pi_{w_t})| \\ &\geq c_i + \beta - J_i(\pi^*) - |\bar{f}_i(w_t) - f_i(\pi_{w_t})| \\ &\geq \beta - \|Q_{\pi_{w_t}}^i - \bar{Q}_t^i\|_2. \end{aligned} \quad (4.18)$$

Substituting equation 4.18 into equation 4.17 gives

$$\begin{aligned}
& \sum_{t \in \mathcal{N}_0} \sum_{i=1}^m \lambda_t^i (f_i(\pi^*) - f_i(\pi_{w_t})) + \beta \sum_{i=1}^p |\mathcal{N}_i| - \sum_{i=m+1}^{m+p} \sum_{t \in \mathcal{N}_i} \|Q_{\pi_{w_t}}^i - \bar{Q}_t^i\|_2 \\
& \leq \frac{1}{\eta} \mathbb{E}_{s \sim \nu^*} D_{\text{KL}}(\pi^* || \pi_{w_0}) + \frac{2\eta c_{\max}^2 |\mathcal{S}| |\mathcal{A}|}{(1-\gamma)^4} \\
& \quad \left(\sum_{t \in \mathcal{N}_0} \left(\sum_{i=1}^m \lambda_t^i \right)^2 + T - |\mathcal{N}_0| \right) + \frac{3}{(1-\gamma)^2} \\
& \quad \left(\sum_{t \in \mathcal{N}_0} \sum_{i=1}^m \lambda_t^i \|Q_{\pi_{w_t}}^i - \bar{Q}_t^i\|_2 + \sum_{i=m+1}^{m+p} \sum_{t \in \mathcal{N}_i} \|Q_{\pi_{w_t}}^i - \bar{Q}_t^i\|_2 \right),
\end{aligned}$$

which implies

$$\begin{aligned}
& \sum_{t \in \mathcal{N}_0} \sum_{i=1}^m \lambda_t^i (f_i(\pi^*) - f_i(\pi_{w_t})) + \beta \sum_{i=1}^p |\mathcal{N}_i| \\
& \leq \frac{1}{\eta} \mathbb{E}_{s \sim \nu^*} D_{\text{KL}}(\pi^* || \pi_{w_0}) + \frac{2\eta c_{\max}^2 |\mathcal{S}| |\mathcal{A}|}{(1-\gamma)^4} \\
& \quad \left(\sum_{t \in \mathcal{N}_0} \left(\sum_{i=1}^m \lambda_t^i \right)^2 + T - |\mathcal{N}_0| \right) + \frac{4}{(1-\gamma)^2} \\
& \quad \left(\sum_{t \in \mathcal{N}_0} \sum_{i=1}^m \lambda_t^i \|Q_{\pi_{w_t}}^i - \bar{Q}_t^i\|_2 + \sum_{i=m+1}^{m+p} \sum_{t \in \mathcal{N}_i} \|Q_{\pi_{w_t}}^i - \bar{Q}_t^i\|_2 \right). \tag{4.19}
\end{aligned}$$

We then first verify item 1. If $\mathcal{N}_0 = \emptyset$, then $\sum_{i=1}^p |\mathcal{N}_i| = T$, and equation 4.19 implies that

$$\begin{aligned}
\beta T & \leq \frac{1}{\eta} \mathbb{E}_{s \sim \nu^*} D_{\text{KL}}(\pi^* || \pi_{w_0}) + \frac{2\eta c_{\max}^2 |\mathcal{S}| |\mathcal{A}|}{(1-\gamma)^4} \left(\sum_{t \in \mathcal{N}_0} \left(\sum_{i=1}^m \lambda_t^i \right)^2 \right. \\
& \quad \left. + T - |\mathcal{N}_0| \right) + \frac{4}{(1-\gamma)^2} \left(\sum_{t \in \mathcal{N}_0} \sum_{i=1}^m \lambda_t^i \|Q_{\pi_{w_t}}^i - \bar{Q}_t^i\|_2 \right. \\
& \quad \left. + \sum_{i=m+1}^{m+p} \sum_{t \in \mathcal{N}_i} \|Q_{\pi_{w_t}}^i - \bar{Q}_t^i\|_2 \right),
\end{aligned}$$

which contradicts equation 4.13. Thus, we must have $\mathcal{N}_0 \neq \emptyset$.

We then proceed to verify item 2. If $\sum_{t \in \mathcal{N}_0} \sum_{i=1}^m \lambda_t^i (f_i(\pi^*) - f_i(\pi_{w_t})) \leq 0$, then (b) in item 2 holds. If $\sum_{t \in \mathcal{N}_0} \sum_{i=1}^m \lambda_t^i (f_i(\pi^*) - f_i(\pi_{w_t})) + \beta \sum_{i=1}^p |\mathcal{N}_i| \geq 0$ and suppose that $|\mathcal{N}_0| < T/2$, i.e., $\sum_{i=1}^p |\mathcal{N}_i| \geq T/2$, then equation 4.19 implies that

$$\begin{aligned}
\frac{1}{2} \beta T & \leq \beta \sum_{i=1}^p |\mathcal{N}_i| \leq \frac{1}{\eta} \mathbb{E}_{s \sim \nu^*} D_{\text{KL}}(\pi^* || \pi_{w_0}) + \frac{2\eta c_{\max}^2 |\mathcal{S}| |\mathcal{A}|}{(1-\gamma)^4} \left(\sum_{t \in \mathcal{N}_0} \left(\sum_{i=1}^m \lambda_t^i \right)^2 + T - |\mathcal{N}_0| \right) \\
& \quad + \frac{4}{(1-\gamma)^2} \left(\sum_{t \in \mathcal{N}_0} \sum_{i=1}^m \lambda_t^i \|Q_{\pi_{w_t}}^i - \bar{Q}_t^i\|_2 + \sum_{i=m+1}^{m+p} \sum_{t \in \mathcal{N}_i} \|Q_{\pi_{w_t}}^i - \bar{Q}_t^i\|_2 \right),
\end{aligned}$$

which contradicts equation 4.13. Hence, (a) in item 2 holds. $\square$

Theorem 4. For a given number of iterations T of CR-MOPO algorithm, with the choices of $\eta = \frac{(1-\gamma)^2}{r_{\max} m B_1} \sqrt{\frac{\mathbb{E}_{s \sim \nu^*} D_{\text{KL}}(\pi^* || \pi_{w_0})}{|\mathcal{S}||\mathcal{A}|T}}$ and $\beta = \frac{4mB_1 \sqrt{|\mathcal{S}||\mathcal{A}|}}{(1-\gamma)^2 \sqrt{T}} (r_{\max} \sqrt{\mathbb{E}_{s \sim \nu^*} D_{\text{KL}}(\pi^* || \pi_{w_0})} + 2)$, with a probability at least $1 - \delta$, we have

$$\begin{aligned} \min_{\lambda^* \in S_m} (\lambda^{*\top} F(\pi^*) - \lambda^{*\top} F(\pi_{w_{\text{out}}})) &\leq \frac{4mB_1 \sqrt{|\mathcal{S}||\mathcal{A}|}}{B_2(1-\gamma)^2 \sqrt{T}} (r_{\max} \sqrt{\mathbb{E}_{s \sim \nu^*} D_{\text{KL}}(\pi^* || \pi_{w_0})} + 2), \quad (4.20) \\ f_i(\pi_{w_{\text{out}}}) - c_i &\leq \frac{4mB_1 \sqrt{|\mathcal{S}||\mathcal{A}|}}{(1-\gamma)^2 \sqrt{T}} \left(r_{\max} \sqrt{\mathbb{E}_{s \sim \nu^*} D_{\text{KL}}(\pi^* || \pi_{w_0})} + 2 + \frac{\sqrt{(1-\gamma)^5}}{2mB_1} \right). \end{aligned}$$

Proof. First we show that the given values for η and β satisfy Lemma 5 as follows,

$$\begin{aligned} &\frac{2}{\eta T} \mathbb{E}_{s \sim \nu^*} D_{\text{KL}}(\pi^* || \pi_{w_0}) + \frac{4\eta c_{\max}^2 |\mathcal{S}||\mathcal{A}|}{(1-\gamma)^4 T} \\ &\quad \left(\sum_{t \in \mathcal{N}_0} \left(\sum_{i=1}^m \lambda_t^i \right)^2 + T - |\mathcal{N}_0| \right) + \frac{8}{(1-\gamma)^2 T} \\ &\quad \left(\sum_{t \in \mathcal{N}_0} \sum_{i=1}^m \lambda_t^i \|Q_{\pi_{w_t}}^i - \bar{Q}_t^i\|_2 + \sum_{i=m+1}^{m+p} \sum_{t \in \mathcal{N}_i} \|Q_{\pi_{w_t}}^i - \bar{Q}_t^i\|_2 \right) \\ &\leq \frac{2}{\eta T} \mathbb{E}_{s \sim \nu^*} D_{\text{KL}}(\pi^* || \pi_{w_0}) + \frac{4\eta c_{\max}^2 |\mathcal{S}||\mathcal{A}|}{(1-\gamma)^4} (m^2 B_1^2 + 1) \\ &\quad + \frac{8mB_1}{(1-\gamma)^2 T} \left(\sum_{t \in \mathcal{N}_0} \|Q_{\pi_{w_t}}^i - \bar{Q}_t^i\|_2 \right) \\ &\leq \frac{2}{\eta T} \mathbb{E}_{s \sim \nu^*} D_{\text{KL}}(\pi^* || \pi_{w_0}) + \frac{4\eta c_{\max}^2 |\mathcal{S}||\mathcal{A}|}{(1-\gamma)^4} (m^2 B_1^2 + 1) \\ &\quad + \frac{8mB_1 \sqrt{|\mathcal{S}||\mathcal{A}|}}{(1-\gamma)^2 \sqrt{T}} \\ &\leq \frac{2}{\eta T} \mathbb{E}_{s \sim \nu^*} D_{\text{KL}}(\pi^* || \pi_{w_0}) + \frac{4\eta c_{\max}^2 |\mathcal{S}||\mathcal{A}|}{(1-\gamma)^4} (m^2 B_1^2 + 1) \\ &\quad + \frac{8mB_1 \sqrt{|\mathcal{S}||\mathcal{A}|}}{(1-\gamma)^2 \sqrt{T}} \\ &< \frac{4r_{\max} m B_1 \sqrt{|\mathcal{S}||\mathcal{A}| \mathbb{E}_{s \sim \nu^*} D_{\text{KL}}(\pi^* || \pi_{w_0})}}{(1-\gamma)^2 \sqrt{T}} + \frac{8mB_1 \sqrt{|\mathcal{S}||\mathcal{A}|}}{(1-\gamma)^2 \sqrt{T}} \\ &= \beta, \end{aligned} \quad (4.21)$$

where the last inequality follows from $\eta = \frac{(1-\gamma)^2}{r_{\max} m B_1} \sqrt{\frac{\mathbb{E}_{s \sim \nu^*} D_{\text{KL}}(\pi^* || \pi_{w_0})}{|\mathcal{S}||\mathcal{A}|T}}$. This verifies that the condition in Lemma 5 is satisfied.

We now consider the convergence rate of the multi-objective optimization. By the property

for the min operator, it holds that

$$\begin{aligned}
& \min_{\lambda^* \in S_m} (\lambda^{*\top} F(\pi_{w_{\text{out}}}) - \lambda^{*\top} F(\pi^*)) \\
& \leq \frac{\sum_{i=1}^m \lambda_{\text{out}}^i f^i(\pi_{\text{out}})}{\sum_{i=1}^m \lambda_{\text{out}}^i} - \frac{\sum_{i=1}^m \lambda_{\text{out}}^i f^i(\pi^*)}{\sum_{i=1}^m \lambda_{\text{out}}^i} \\
& \leq \frac{1}{B_2} \sum_{i=1}^m \lambda_{\text{out}}^i f^i(\pi_{\text{out}}) - \sum_{i=1}^m \lambda_{\text{out}}^i f^i(\pi^*) \\
& \leq \frac{1}{B_2 |\mathcal{N}_0|} \sum_{t \in \mathcal{N}_0} \sum_{i=1}^m \lambda_t^i (f_i(\pi^*) - f_i(\pi_{w_t})),
\end{aligned}$$

where the second inequality follows from $\sum_{i=1}^m \lambda_t^i \geq B_2$ for $t = 1, \dots, T$ in Assumption 1. If $\sum_{t \in \mathcal{N}_0} \sum_{i=1}^m \lambda_t^i (f_i(\pi^*) - f_i(\pi_{w_t})) \leq 0$, then we have $\min_{\lambda^* \in S_m} (\lambda^{*\top} F(\pi_{w_{\text{out}}}) - \lambda^{*\top} F(\pi^*)) \leq 0$. If $\sum_{t \in \mathcal{N}_0} \sum_{i=1}^m \lambda_t^i (f_i(\pi^*) - f_i(\pi_{w_t})) \geq 0$, we have $|\mathcal{N}_0| \geq \frac{T}{2}$, which implies the following convergence rate

$$\begin{aligned}
& \min_{\lambda^* \in S_m} (\lambda^{*\top} F(\pi_{w_{\text{out}}}) - \lambda^{*\top} F(\pi^*)) \\
& \leq \frac{4r_{\max} m B_1 \sqrt{|\mathcal{S}| |\mathcal{A}| \mathbb{E}_{s \sim \nu^*} D_{\text{KL}}(\pi^* || \pi_{w_0})}}{(1-\gamma)^2 \sqrt{T}} + \frac{8m B_1 \sqrt{m} |\mathcal{S}| |\mathcal{A}|}{(1-\gamma)^3 \sqrt{T}}.
\end{aligned}$$

We then proceed to bound the constraints violation. For any $i \in \{m+1, \dots, m+p\}$, it holds that

$$\begin{aligned}
f_i(\pi_{w_{\text{out}}}) - c_i &= \frac{1}{|\mathcal{N}_0|} \sum_{t \in \mathcal{N}_0} f_i(\pi_{w_t}) - c_i \\
&\leq \frac{1}{|\mathcal{N}_0|} \sum_{t \in \mathcal{N}_0} (\bar{f}_i(\theta_t^i) - c_i) + \frac{1}{|\mathcal{N}_0|} \sum_{t \in \mathcal{N}_0} (f_i(\pi_{w_t}) - \bar{f}_i(\theta_t^i)) \\
&\leq \beta + \frac{1}{|\mathcal{N}_0|} \sum_{t=0}^{T-1} (f_i(\pi_{w_t}) - \bar{f}_i(\theta_t^i)) \\
&\leq \beta + \frac{2}{T} \sum_{t=0}^{T-1} f_i(\pi_{w_t}) - \bar{f}_i(\theta_t^i) \\
&\leq \beta + \frac{2}{T} \sum_{t=0}^{T-1} \|Q_{\pi_{w_t}}^i - \bar{Q}_t^i\|_2.
\end{aligned}$$

Under the condition defined in equation 4.13, we have $\frac{2}{T} \sum_{t=0}^{T-1} \|Q_{\pi_{w_t}}^i - \bar{Q}_t^i\|_2 \leq \frac{2\sqrt{(1-\gamma)|\mathcal{S}||\mathcal{A}|}}{\sqrt{T}}$, which gives us convergence rates given in the statement. $\square$

4.8.2 Unbiased Q-Estimation Estimator

Lemma 6. For a given $t \in [T]$, we have the following

$$\left\| \mathbb{E} \left[\sum_{i=1}^m \lambda^i (Q_{\pi_{w_t}}^i - \bar{Q}_t^i) \right] \right\|_2 \leq \sqrt{\mathbb{V}(\lambda) \sum_{i=1}^m \mathbb{V}(\bar{Q}_t^i)}.$$

Proof.

$$\begin{aligned}
& \left\| \mathbb{E} \left[\sum_{i=1}^m \lambda^i (Q_{\pi_{w_t}}^i - \bar{Q}_t^i) \right] \right\|_2 \\
&= \left\| \mathbb{E} \left[\sum_{i=1}^m (\lambda^i - \mathbb{E}[\lambda^i]) (Q_{\pi_{w_t}}^i - \bar{Q}_t^i) \right] + \sum_{i=1}^m \mathbb{E}[\lambda^i] \mathbb{E}[(Q_{\pi_{w_t}}^i - \bar{Q}_t^i)] \right\|_2 \\
&= \left\| \mathbb{E} \left[\sum_{i=1}^m (\lambda^i - \mathbb{E}[\lambda^i]) (Q_{\pi_{w_t}}^i - \bar{Q}_t^i) \right] \right\|_2 \\
&\leq \mathbb{E} \left[\left\| \sum_{i=1}^m (\lambda^i - \mathbb{E}[\lambda^i]) (Q_{\pi_{w_t}}^i - \bar{Q}_t^i) \right\|_2 \right] \\
&\leq \mathbb{E} \left[\left\| \sum_{i=1}^m (\lambda^i - \mathbb{E}[\lambda^i]) (Q_{\pi_{w_t}}^i - \bar{Q}_t^i) \right\|_2 \right] \\
&\leq \mathbb{E} \left[\sum_{i=1}^m \left\| (\lambda^i - \mathbb{E}[\lambda^i]) (Q_{\pi_{w_t}}^i - \bar{Q}_t^i) \right\|_2 \right] \\
&\leq \mathbb{E} \left[\sum_{i=1}^m \|\lambda^i - \mathbb{E}[\lambda^i]\|_2 \|Q_{\pi_{w_t}}^i - \bar{Q}_t^i\|_2 \right] \\
&\leq \sum_{i=1}^m \mathbb{E} \left[\frac{\beta_t}{2} \|\lambda^i - \mathbb{E}[\lambda^i]\|_2^2 + \frac{1}{2\beta_t} \|Q_{\pi_{w_t}}^i - \bar{Q}_t^i\|_2^2 \right] \\
&= \frac{\beta_t}{2} \mathbb{V}(\lambda) + \frac{1}{2\beta_t} \sum_{i=1}^m \mathbb{V}(\bar{Q}_t^i) \\
&\leq \sqrt{\mathbb{V}(\lambda) \sum_{i=1}^m \mathbb{V}(\bar{Q}_t^i)}.
\end{aligned}$$

□

Lemma 7 (Lemma 2 of [292]). *Under Assumption 1, the variance of λ_t in Algorithm 3 is bounded $\mathbb{V}[\lambda_t] \leq m^2 B_1^2 (1 - \alpha_t)^2$.*

Lemma 8. *If*

$$\begin{aligned}
\beta &> \frac{2}{\eta T} \mathbb{E}_{s \sim \gamma^*} D_{\text{KL}}(\pi^* || \pi_{w_0}) + \frac{4\eta c_{\max}^2 |S| |\mathcal{A}|}{(1 - \gamma)^4 T} \\
&\quad \left(\mathbb{E} \left[\sum_{t \in \mathcal{N}_0} \left(\sum_{i=1}^m \lambda_t^i \right)^2 \right] + T - |\mathcal{N}_0| \right) \\
&\quad + \frac{8}{(1 - \gamma)^2 T} \left(\sum_{t \in \mathcal{N}_0} \left\| \mathbb{E} \left[\sum_{i=1}^m \lambda_t^i (Q_{\pi_{w_t}}^i - \bar{Q}_t^i) \right] \right\|_2 \right) \\
&\quad + \sum_{i=m+1}^{m+p} \sum_{t \in \mathcal{N}_i} \left\| \mathbb{E} [Q_{\pi_{w_t}}^i - \bar{Q}_t^i] \right\|_2,
\end{aligned} \tag{4.22}$$

then we have the following holds:

1. $\mathcal{N}_0 \neq \emptyset$,
2. One of the following two statements must hold,

- (a) $|\mathcal{N}_0| \geq \frac{T}{2}$,
 (b) $\mathbb{E} \left[\sum_{t \in \mathcal{N}_0} \sum_{i=1}^m \lambda_t^i (f_i(\pi^*) - f_i(\pi_{w_t})) \right] \leq 0$.

Proof. When $t \in \mathcal{N}_0$, from Lemma 3 we have

$$\begin{aligned}
 & \mathbb{E} \left[\sum_{i=1}^m \lambda^i (f_i(\pi^*) - f_i(\pi_{w_t})) \right] \\
 & \leq \frac{1}{\eta} \mathbb{E}_{s \sim \nu^*} (\mathbb{E} [D_{\text{KL}}(\pi^* || \pi_{w_t}) - D_{\text{KL}}(\pi^* || \pi_{w_{t+1}})]) \\
 & \quad + \frac{2\eta c_{\max}^2 |\mathcal{S}| |\mathcal{A}|}{(1-\gamma)^4} \mathbb{E} \left[\left(\sum_{i=1}^m \lambda^i \right)^2 \right] \\
 & \quad + \frac{3}{(1-\gamma)^2} \left\| \mathbb{E} \left[\sum_{i=1}^m \lambda^i (Q_{\pi_{w_t}}^i - \bar{Q}_t^i) \right] \right\|_2.
 \end{aligned} \tag{4.23}$$

Similarly, when $t \in \mathcal{N}_i (i \neq 0)$ we have

$$\begin{aligned}
 & \mathbb{E} [f_i(\pi_{w_t}) - f_i(\pi^*)] \\
 & \leq \frac{1}{\eta} \mathbb{E}_{s \sim \nu^*} (\mathbb{E} [D_{\text{KL}}(\pi^* || \pi_{w_t}) - D_{\text{KL}}(\pi^* || \pi_{w_{t+1}})]) \\
 & \quad + \frac{2\eta c_{\max}^2 |\mathcal{S}| |\mathcal{A}|}{(1-\gamma)^4} + \frac{3}{(1-\gamma)^2} \left\| \mathbb{E} [Q_{\pi_{w_t}}^i - \bar{Q}_t^i] \right\|_2.
 \end{aligned} \tag{4.24}$$

Taking the summation of equation 4.23 and equation 4.24 from $t = 0$ to $T - 1$ gives

$$\begin{aligned}
 & \mathbb{E} \left[\sum_{t \in \mathcal{N}_0} \sum_{i=1}^m \lambda_t^i (f_i(\pi^*) - f_i(\pi_{w_t})) \right. \\
 & \quad \left. + \sum_{i=1}^p \sum_{t \in \mathcal{N}_i} (f_i(\pi_{w_t}) - f_i(\pi^*)) \right] \\
 & \leq \frac{1}{\eta} \mathbb{E}_{s \sim \nu^*} D_{\text{KL}}(\pi^* || \pi_{w_0}) \\
 & \quad + \frac{2\eta c_{\max}^2 |\mathcal{S}| |\mathcal{A}|}{(1-\gamma)^4} \left(\mathbb{E} \left[\sum_{t \in \mathcal{N}_0} \left(\sum_{i=1}^m \lambda_t^i \right)^2 \right] + T - |\mathcal{N}_0| \right) \\
 & \quad + \frac{3}{(1-\gamma)^2} \left(\sum_{t \in \mathcal{N}_0} \left\| \mathbb{E} \left[\sum_{i=1}^m \lambda^i (Q_{\pi_{w_t}}^i - \bar{Q}_t^i) \right] \right\|_2 \right. \\
 & \quad \left. + \sum_{i=m+1}^{m+p} \sum_{t \in \mathcal{N}_i} \left\| \mathbb{E} [Q_{\pi_{w_t}}^i - \bar{Q}_t^i] \right\|_2 \right).
 \end{aligned} \tag{4.25}$$

Note that when $t \in \mathcal{N}_i (i \neq 0)$, we have $\bar{f}_i(w_t) > c_i + \beta$ (line 9 in Algorithm 1), which implies that

$$\begin{aligned}
 f_i(\pi_{w_t}) - f_i(\pi^*) & \geq \bar{f}_i(w_t) - f_i(\pi^*) - |\bar{f}_i(w_t) - f_i(\pi_{w_t})| \\
 & \geq c_i + \beta - J_i(\pi^*) - |\bar{f}_i(w_t) - f_i(\pi_{w_t})| \\
 & \geq \beta - \left\| Q_{\pi_w}^i - \bar{Q}_t^i \right\|_2.
 \end{aligned} \tag{4.26}$$

Substituting equation 4.26 into equation 4.25 gives

$$\begin{aligned}
& \mathbb{E} \left[\sum_{t \in \mathcal{N}_0} \sum_{i=1}^m \lambda_t^i (f_i(\pi^*) - f_i(\pi_{w_t})) + \beta \sum_{i=1}^p |\mathcal{N}_i| - \sum_{i=m+1}^{m+p} \sum_{t \in \mathcal{N}_i} \|Q_{\pi_w}^i - \bar{Q}_t^i\|_2 \right] \\
& \leq \frac{1}{\eta} \mathbb{E}_{s \sim \nu^*} D_{\text{KL}}(\pi^* || \pi_{w_0}) \\
& \quad + \frac{2\eta c_{\max}^2 |S| |\mathcal{A}|}{(1-\gamma)^4} \left(\mathbb{E} \left[\sum_{t \in \mathcal{N}_0} \left(\sum_{i=1}^m \lambda_t^i \right)^2 \right] + T - |\mathcal{N}_0| \right) \\
& \quad + \frac{3}{(1-\gamma)^2} \left(\sum_{t \in \mathcal{N}_0} \left\| \mathbb{E} \left[\sum_{i=1}^m \lambda^i (Q_{\pi_{w_t}}^i - \bar{Q}_t^i) \right] \right\|_2 + \sum_{i=m+1}^{m+p} \sum_{t \in \mathcal{N}_i} \left\| \mathbb{E} [Q_{\pi_{w_t}}^i - \bar{Q}_t^i] \right\|_2 \right),
\end{aligned}$$

which implies

$$\begin{aligned}
& \mathbb{E} \left[\sum_{t \in \mathcal{N}_0} \sum_{i=1}^m \lambda_t^i (f_i(\pi^*) - f_i(\pi_{w_t})) + \beta \sum_{i=1}^p |\mathcal{N}_i| \right] \\
& \leq \frac{1}{\eta} \mathbb{E}_{s \sim \nu^*} D_{\text{KL}}(\pi^* || \pi_{w_0}) + \frac{2\eta c_{\max}^2 |S| |\mathcal{A}|}{(1-\gamma)^4} \left(\mathbb{E} \left[\sum_{t \in \mathcal{N}_0} \left(\sum_{i=1}^m \lambda_t^i \right)^2 \right] + T - |\mathcal{N}_0| \right) \\
& \quad + \frac{4}{(1-\gamma)^2} \left(\sum_{t \in \mathcal{N}_0} \left\| \mathbb{E} \left[\sum_{i=1}^m \lambda^i (Q_{\pi_{w_t}}^i - \bar{Q}_t^i) \right] \right\|_2 + \sum_{i=m+1}^{m+p} \sum_{t \in \mathcal{N}_i} \left\| \mathbb{E} [Q_{\pi_{w_t}}^i - \bar{Q}_t^i] \right\|_2 \right).
\end{aligned}$$

We then first verify item 1. If $\mathcal{N}_0 = \emptyset$, then $\sum_{i=1}^p |\mathcal{N}_i| = T$, and equation 4.27 implies that

$$\begin{aligned}
\beta T & \leq \frac{1}{\eta} \mathbb{E}_{s \sim \nu^*} D_{\text{KL}}(\pi^* || \pi_{w_0}) + \frac{2\eta c_{\max}^2 |S| |\mathcal{A}|}{(1-\gamma)^4} \left(\mathbb{E} \left[\sum_{t \in \mathcal{N}_0} \left(\sum_{i=1}^m \lambda_t^i \right)^2 \right] + T - |\mathcal{N}_0| \right) \\
& \quad + \frac{4}{(1-\gamma)^2} \left(\sum_{t \in \mathcal{N}_0} \left\| \mathbb{E} \left[\sum_{i=1}^m \lambda^i (Q_{\pi_{w_t}}^i - \bar{Q}_t^i) \right] \right\|_2 + \sum_{i=m+1}^{m+p} \sum_{t \in \mathcal{N}_i} \left\| \mathbb{E} [Q_{\pi_{w_t}}^i - \bar{Q}_t^i] \right\|_2 \right),
\end{aligned}$$

which contradicts equation 4.22. Thus, we must have $\mathcal{N}_0 \neq \emptyset$.

We then proceed to verify item 2. If $\mathbb{E} \left[\sum_{t \in \mathcal{N}_0} \sum_{i=1}^m \lambda_t^i (f_i(\pi^*) - f_i(\pi_{w_t})) \right] \leq 0$, then (b) in item 2 holds. If $\mathbb{E} \left[\sum_{t \in \mathcal{N}_0} \sum_{i=1}^m \lambda_t^i (f_i(\pi^*) - f_i(\pi_{w_t})) + \beta \sum_{i=1}^p |\mathcal{N}_i| \right] \geq 0$ and suppose that $|\mathcal{N}_0| < T/2$, i.e., $\sum_{i=1}^p |\mathcal{N}_i| \geq T/2$, then equation 4.27 implies that

$$\begin{aligned}
\frac{1}{2} \beta T & \leq \beta \sum_{i=1}^p |\mathcal{N}_i| \leq \frac{1}{\eta} \mathbb{E}_{s \sim \nu^*} D_{\text{KL}}(\pi^* || \pi_{w_0}) + \frac{2\eta c_{\max}^2 |S| |\mathcal{A}|}{(1-\gamma)^4} \left(\mathbb{E} \left[\sum_{t \in \mathcal{N}_0} \left(\sum_{i=1}^m \lambda_t^i \right)^2 \right] + T - |\mathcal{N}_0| \right) \\
& \quad + \frac{4}{(1-\gamma)^2} \left(\sum_{t \in \mathcal{N}_0} \left\| \mathbb{E} \left[\sum_{i=1}^m \lambda^i (Q_{\pi_{w_t}}^i - \bar{Q}_t^i) \right] \right\|_2 + \sum_{i=m+1}^{m+p} \sum_{t \in \mathcal{N}_i} \left\| \mathbb{E} [Q_{\pi_{w_t}}^i - \bar{Q}_t^i] \right\|_2 \right),
\end{aligned}$$

which contradicts equation 4.22. Hence, (a) in item 2 holds. $\square$

Theorem 5. For a given number of iterations T of CR-MOPO algorithm, with the choices of

$\eta = \frac{(1-\gamma)^2}{r_{\max} m B_1} \sqrt{\frac{\mathbb{E}_{s \sim \nu^*} D_{\text{KL}}(\pi^* || \pi_{w_0})}{|S||A|T}}$ and $\beta = \frac{4mB_1 \sqrt{|S||A|}}{(1-\gamma)^2 \sqrt{T}} (r_{\max} \sqrt{\mathbb{E}_{s \sim \nu^*} D_{\text{KL}}(\pi^* || \pi_{w_0})} + 1)$, we have

$$\mathbb{E} \left[\min_{\lambda^* \in S_m} (\lambda^{*\top} F(\pi^*) - \lambda^{*\top} F(\pi_{w_{out}})) \right] \leq \frac{4mB_1 \sqrt{|S||A|}}{B_2(1-\gamma)^2 \sqrt{T}} (r_{\max} \sqrt{\mathbb{E}_{s \sim \nu^*} D_{\text{KL}}(\pi^* || \pi_{w_0})} + 1), \quad (4.27)$$

$$\mathbb{E}[f_i(\pi_{w_{out}})] - c_i \leq \frac{4mB_1 \sqrt{|S||A|}}{(1-\gamma)^2 \sqrt{T}} (r_{\max} \sqrt{\mathbb{E}_{s \sim \nu^*} D_{\text{KL}}(\pi^* || \pi_{w_0})} + 1). \quad (4.28)$$

Proof. First we show that the given values for η and β satisfy Lemma 8 as follows,

$$\begin{aligned} & \frac{2}{\eta T} \mathbb{E}_{s \sim \nu^*} D_{\text{KL}}(\pi^* || \pi_{w_0}) + \frac{4\eta c_{\max}^2 |S||A|}{(1-\gamma)^4 T} \left(\mathbb{E} \left[\sum_{t \in \mathcal{N}_0} \left(\sum_{i=1}^m \lambda_t^i \right)^2 \right] + T - |\mathcal{N}_0| \right) \\ & + \frac{8}{(1-\gamma)^2 T} \left(\sum_{t \in \mathcal{N}_0} \left\| \mathbb{E} \left[\sum_{i=1}^m \lambda^i (Q_{\pi_{w_t}}^i - \bar{Q}_t^i) \right] \right\|_2 + \sum_{i=m+1}^{m+p} \sum_{t \in \mathcal{N}_i} \left\| \mathbb{E} [Q_{\pi_{w_t}}^i - \bar{Q}_t^i] \right\|_2 \right) \\ & \leq \frac{2}{\eta T} \mathbb{E}_{s \sim \nu^*} D_{\text{KL}}(\pi^* || \pi_{w_0}) + \frac{4\eta c_{\max}^2 |S||A|}{(1-\gamma)^4} (m^2 B_1^2 + 1) + \frac{8}{(1-\gamma)^2 T} \left(\sum_{t \in \mathcal{N}_0} \left\| \mathbb{E} \left[\sum_{i=1}^m \lambda^i (Q_{\pi_{w_t}}^i - \bar{Q}_t^i) \right] \right\|_2 \right) \\ & \leq \frac{2}{\eta T} \mathbb{E}_{s \sim \nu^*} D_{\text{KL}}(\pi^* || \pi_{w_0}) + \frac{4\eta c_{\max}^2 |S||A|}{(1-\gamma)^4} (m^2 B_1^2 + 1) + \frac{8}{(1-\gamma)^2 T} \left(\sum_{t \in \mathcal{N}_0} \sqrt{\mathbb{V}(\lambda) \sum_{i=1}^m \mathbb{V}(\bar{Q}_t^i)} \right) \\ & \leq \frac{2}{\eta T} \mathbb{E}_{s \sim \nu^*} D_{\text{KL}}(\pi^* || \pi_{w_0}) + \frac{4\eta c_{\max}^2 |S||A|}{(1-\gamma)^4} (m^2 B_1^2 + 1) + \frac{8mB_1}{(1-\gamma)^2 T} \left(\sum_{t \in \mathcal{N}_0} (1 - \alpha_t) \sqrt{\sum_{i=1}^m \mathbb{V}(\bar{Q}_t^i)} \right) \\ & \leq \frac{2}{\eta T} \mathbb{E}_{s \sim \nu^*} D_{\text{KL}}(\pi^* || \pi_{w_0}) + \frac{4\eta c_{\max}^2 |S||A|}{(1-\gamma)^4} (m^2 B_1^2 + 1) + \frac{4mB_1 \sqrt{m} |S||A|}{(1-\gamma)^3 T} \sum_{t \in \mathcal{N}_0} (1 - \alpha_t) \\ & \leq \frac{2}{\eta T} \mathbb{E}_{s \sim \nu^*} D_{\text{KL}}(\pi^* || \pi_{w_0}) + \frac{4\eta c_{\max}^2 |S||A|}{(1-\gamma)^4} (m^2 B_1^2 + 1) + \frac{4mB_1 \sqrt{m} |S||A|}{(1-\gamma)^3 T} \sum_{t \in [T]} (1 - \alpha_t) \\ & = \frac{2}{\eta T} \mathbb{E}_{s \sim \nu^*} D_{\text{KL}}(\pi^* || \pi_{w_0}) + \frac{4\eta c_{\max}^2 |S||A|}{(1-\gamma)^4} (m^2 B_1^2 + 1) + \frac{4mB_1 \sqrt{m} |S||A|}{(1-\gamma)^3 T} \sum_{t \in [T]} (1 - \alpha_t) \\ & = \frac{2}{\eta T} \mathbb{E}_{s \sim \nu^*} D_{\text{KL}}(\pi^* || \pi_{w_0}) + \frac{4\eta c_{\max}^2 |S||A|}{(1-\gamma)^4} (m^2 B_1^2 + 1) + \frac{4mB_1 \sqrt{|S||A|}}{(1-\gamma)^2 T} \sum_{t \in [T]} \frac{1}{\sqrt{t}} \\ & \leq \frac{2}{\eta T} \mathbb{E}_{s \sim \nu^*} D_{\text{KL}}(\pi^* || \pi_{w_0}) + \frac{4\eta c_{\max}^2 |S||A|}{(1-\gamma)^4} (m^2 B_1^2 + 1) + \frac{4mB_1 \sqrt{|S||A|}}{(1-\gamma)^2 \sqrt{T}} \\ & \leq \frac{2}{\eta T} \mathbb{E}_{s \sim \nu^*} D_{\text{KL}}(\pi^* || \pi_{w_0}) + \frac{4\eta c_{\max}^2 |S||A|}{(1-\gamma)^4} (m^2 B_1^2 + 1) + \frac{4mB_1 \sqrt{|S||A|}}{(1-\gamma)^2 \sqrt{T}} \\ & < \frac{4r_{\max} m B_1 \sqrt{|S||A|} \mathbb{E}_{s \sim \nu^*} D_{\text{KL}}(\pi^* || \pi_{w_0})}{(1-\gamma)^2 \sqrt{T}} + \frac{4mB_1 \sqrt{|S||A|}}{(1-\gamma)^2 \sqrt{T}} \\ & = \frac{4mB_1 \sqrt{|S||A|}}{(1-\gamma)^2 \sqrt{T}} (r_{\max} \sqrt{\mathbb{E}_{s \sim \nu^*} D_{\text{KL}}(\pi^* || \pi_{w_0})} + 1) \\ & = \beta, \end{aligned} \quad (4.29)$$

where the last inequality follows from $\eta = \frac{(1-\gamma)^2}{r_{\max} m B_1} \sqrt{\frac{\mathbb{E}_{s \sim \nu^*} D_{\text{KL}}(\pi^* || \pi_{w_0})}{|S||A|T}}$. This verifies that the condition in Lemma 8 is satisfied.

We now consider the convergence rate of the multi-objective optimization. By the property for the min operator, it holds that

$$\begin{aligned} \mathbb{E} \left[\min_{\lambda^* \in S_m} (\lambda^{*\top} F(\pi_{w_{\text{out}}}) - \lambda^{*\top} F(\pi^*)) \right] &\leq \mathbb{E} \left[\frac{\sum_{i=1}^m \lambda_{\text{out}}^i f^i(\pi_{\text{out}})}{\sum_{i=1}^m \lambda_{\text{out}}^i} - \frac{\sum_{i=1}^m \lambda_{\text{out}}^i f^i(\pi^*)}{\sum_{i=1}^m \lambda_{\text{out}}^i} \right] \\ &\leq \frac{1}{B_2} \mathbb{E} \left[\sum_{i=1}^m \lambda_{\text{out}}^i f^i(\pi_{\text{out}}) - \sum_{i=1}^m \lambda_{\text{out}}^i f^i(\pi^*) \right] \\ &\leq \frac{1}{B_2 |\mathcal{N}_0|} \mathbb{E} \left[\sum_{t \in \mathcal{N}_0} \sum_{i=1}^m \lambda_t^i (f_i(\pi^*) - f_i(\pi_{w_t})) \right], \end{aligned}$$

where the second inequality follows from $\sum_{i=1}^m \lambda_t^i \geq B_2$ for $t = 1, \dots, T$ in Assumption 1. If $\mathbb{E} \left[\sum_{t \in \mathcal{N}_0} \sum_{i=1}^m \lambda_t^i (f_i(\pi^*) - f_i(\pi_{w_t})) \right] \leq 0$, then we have $\mathbb{E} \left[\min_{\lambda^* \in S_m} (\lambda^{*\top} F(\pi_{w_{\text{out}}}) - \lambda^{*\top} F(\pi^*)) \right] \leq 0$. If $\mathbb{E} \left[\sum_{t \in \mathcal{N}_0} \sum_{i=1}^m \lambda_t^i (f_i(\pi^*) - f_i(\pi_{w_t})) \right] \geq 0$, we have $|\mathcal{N}_0| \geq \frac{T}{2}$, which implies the following convergence rate

$$\mathbb{E} \left[\min_{\lambda^* \in S_m} (\lambda^{*\top} F(\pi_{w_{\text{out}}}) - \lambda^{*\top} F(\pi^*)) \right] \leq \frac{4r_{\max} m B_1 \sqrt{|\mathcal{S}| |\mathcal{A}| \mathbb{E}_{s \sim \gamma^*} D_{\text{KL}}(\pi^* || \pi_{w_0})}}{B_2 (1 - \gamma)^2 \sqrt{T}} + \frac{4m B_1 \sqrt{|\mathcal{S}| |\mathcal{A}|}}{B_2 (1 - \gamma)^2 \sqrt{T}}.$$

We then proceed to bound the constraints violation. For any $i \in \{m+1, \dots, m+p\}$, it holds that

$$\begin{aligned} &\mathbb{E} [f_i(\pi_{w_{\text{out}}})] - c_i \\ &= \mathbb{E} \left[\frac{1}{|\mathcal{N}_0|} \sum_{t \in \mathcal{N}_0} f_i(\pi_{w_t}) \right] - c_i \\ &\leq \mathbb{E} \left[\frac{1}{|\mathcal{N}_0|} \sum_{t \in \mathcal{N}_0} (\bar{f}_i(\theta_t^i) - c_i) + \frac{1}{|\mathcal{N}_0|} \sum_{t \in \mathcal{N}_0} (f_i(\pi_{w_t}) - \bar{f}_i(\theta_t^i)) \right] \\ &\leq \beta + \mathbb{E} \left[\frac{1}{|\mathcal{N}_0|} \sum_{t=0}^{T-1} (f_i(\pi_{w_t}) - \bar{f}_i(\theta_t^i)) \right] \\ &\leq \beta + \frac{2}{T} \sum_{t=0}^{T-1} \mathbb{E} [f_i(\pi_{w_t}) - \bar{f}_i(\theta_t^i)] \\ &\leq \beta + \frac{2}{T} \sum_{t=0}^{T-1} \|\mathbb{E} [Q_{\pi_{w_t}}^i - \bar{Q}_t^i]\|_2 \\ &= \beta. \end{aligned}$$

This completes the proof. $\square$

4.9 Experiment Details of Safe Multi-Objective Experiments

4.9.1 Additional Experiments

In Figure 4.4, we present additional experiments that showcase the efficacy of our approach.

4.9.2 Implementation Details of Safe Multi-Objective Reinforcement Learning

Tables 5.7 and 4.2 provide the safety-bound parameters and algorithm parameters used in the study. The server with 40 CPU cores (Intel® Xeon(R) Gold 5218R CPU @ 2.10GHz $\times$ 80) and 1 GTX-970 GPU (NVIDIA GeForce GTX 970/PCIe/SSE2) is used to run the experiments on a Ubuntu 18.04 system.

Environment	value
Safe Multi-Objective HalfCheetah-v4	0.1
Safe Multi-Objective Humanoid-v4	0.9
Safe Multi-Objective Walker-v4	0.03
Safe Multi-Objective Humanoid-dm	1.5
Safe Multi-Objective HalfCheetah-v4-different-limit	0.3
Safe Multi-Objective Hopper-v4	0.03
Safe Multi-Objective Hopper-v4-v2	0.045
Safe Multi-Objective Swimmer-v4	0.049
Safe Multi-Objective Pusher-v4	0.49
Safe Multi-Objective Walker-dm	1.5
Safe Multi-Objective-v4- soft-v1	0.005

Table 4.1: Safety bound of each step used in the Safe Multi-Objective Environments.

Parameters	value	Parameters	value
gamma	0.995	tau	0.97
l2-reg	1e-3	kl	0.05
damping	1e-1	batch-size	16000
epoch	500	episode length	1000
grad-c	0.5	neural network	MLP
hidden layer dim	64	accept ratio	0.1
energy weight	1.0	forward reward weight	1.0

Table 4.2: Key hyparameters used in experiments.

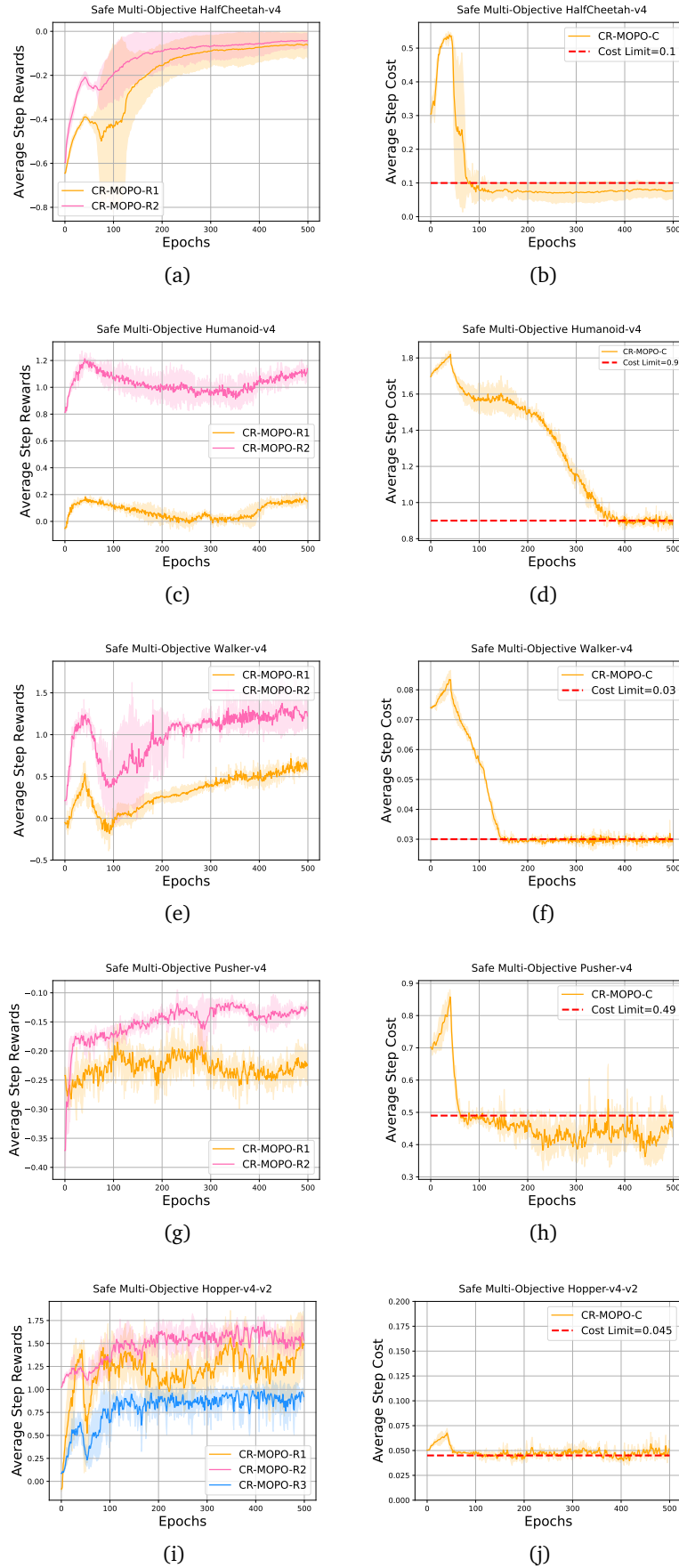


Figure 4.1: CR-MOPO on Safe Multi-Objective MuJoCo environments regarding the reward and safety performance.

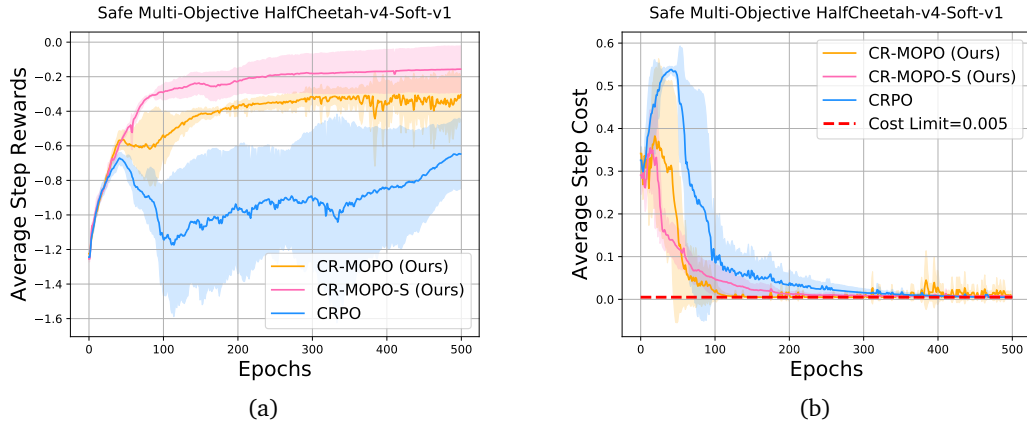


Figure 4.2: (a) and (b) show the comparison results in terms of CR-MOPO, CR-MOPO-S and CRPO [266] on a Safe Multi-Objective MuJoCo environment, Safe Multi-Objective HalfCheetah, the cost limit is 0.005, we start to optimize safety violation after 40 Epochs.

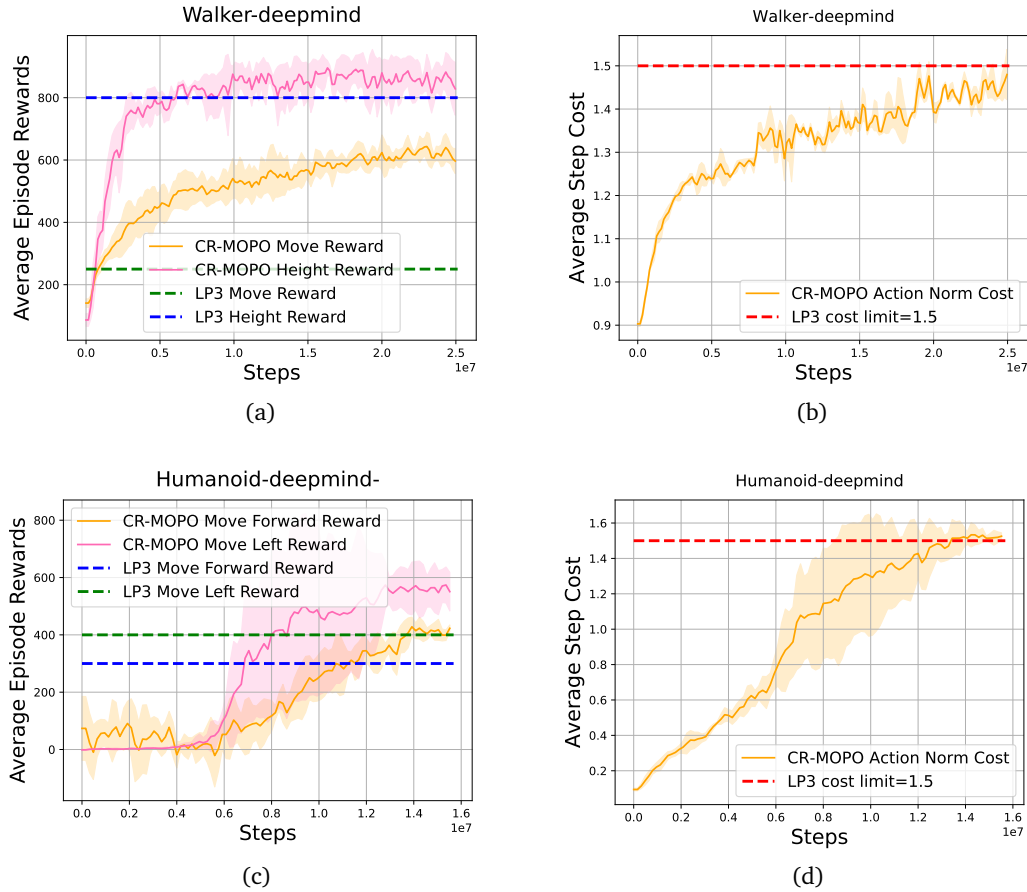


Figure 4.3: Compared with LP3 [107], on Safe Multi-Objective Walker-dm and Safe Multi-Objective Humanoid-dm environments.

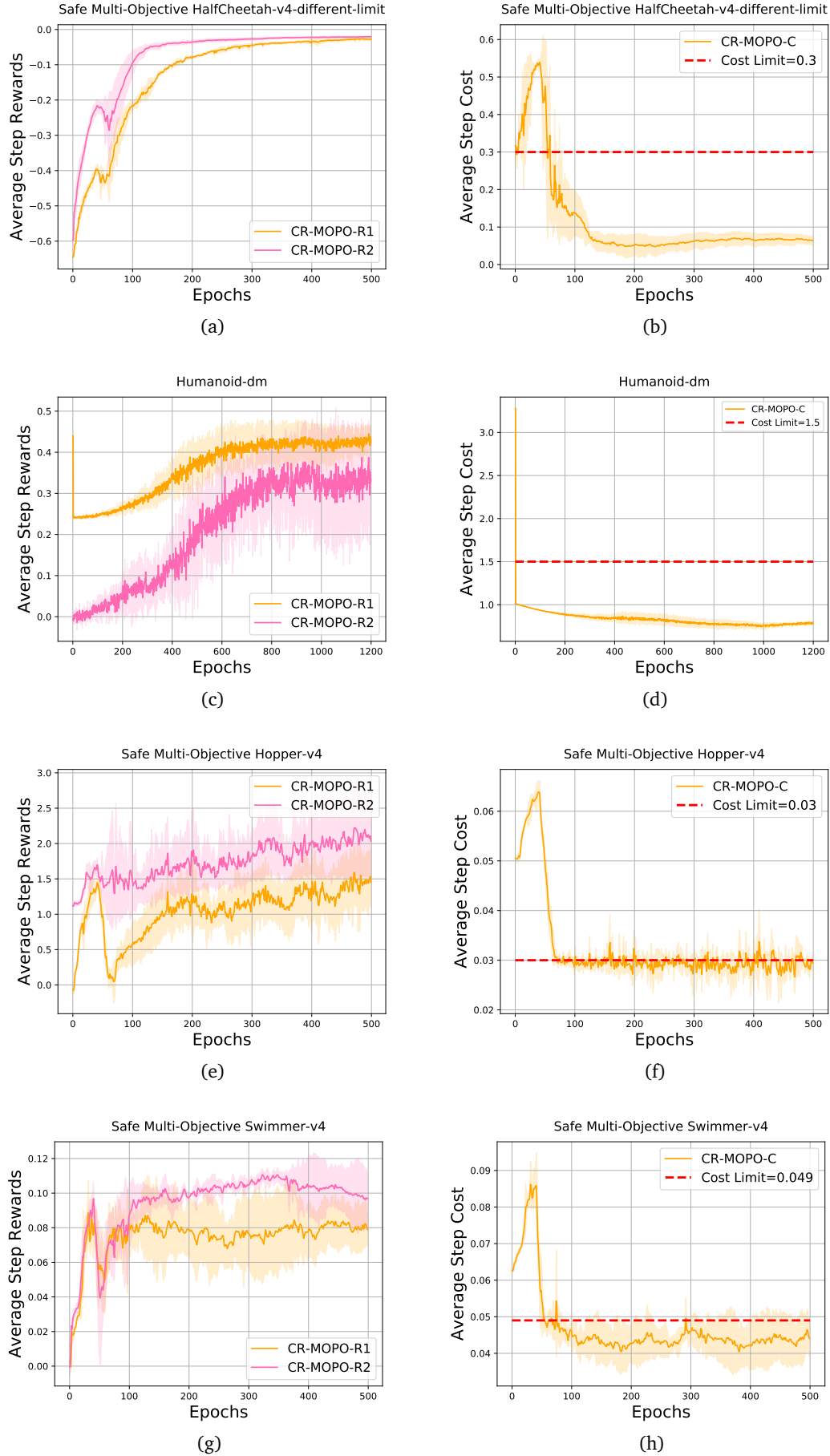


Figure 4.4: More Experiments to evaluate the effectiveness of our method on Safe Multi-Objective MuJoCo environments regarding the reward and safety performance.

5

Safe Multi-Robot Reinforcement Learning

This study addresses the challenge of cooperatively and safely controlling multiple robots in RL applications, a relatively underexplored area in safe MARL. To bridge this research gap, the study focuses on safe MARL for multi-robot control in cooperative tasks, where individual robots must adhere to their own safety constraints while maximizing rewards and ensuring safe multi-robot behaviors. The study formulates the safe MARL problem as a constrained Markov game and theoretically solves it using policy optimization. The proposed algorithm guarantees monotonic reward improvement and consistent adherence to safety constraints. Experimental results demonstrate that the proposed methods achieve SOTA performance and effectively guarantee reward improvement with safety constraint satisfaction when compared to strong baselines.

5.1 Introduction of Safe Multi-Robot Reinforcement Learning

In the past several decades, traditional control methods have played an important role in multi-robot control, e.g. the Lyapunov based method [190], backstepping method [45], and feedback linearization method [76] have been widely adopted in the field. However, these model-based control methods may be hard to generalize to complex tasks with unknown dynamics [23, 40, 104]. On the contrary, in recent years, RL has been successfully applied in many tasks, such as transportation schedule [48], traffic signal control [54], the game of Go [224], autonomous driving [86, 163], finance [3], recommender systems [291] and robotics [40]. The robustness of RL to variety of tasks comes from its experience-driven learning that, while relying on a learner's ability to interact with an environment, does not have to possess all of the environment's underlying information. As such, an RL agent formulates a problem of solving a task via maximisation of the reward signal by taking adequate actions [234]. In combination with expressive deep neural networks that model an agent's policy, these methods have made remarkable advancements in the past decade [144, 216, 218, 267].

However, in the context of practical applications, most RL algorithms share a major limitation—they optimise the agents' policies purely for the sake of reward maximisation, completely ignoring safety considerations. For example, in robot control scenarios, it is essential that a robot does not visit certain states, or must not take certain actions, which can be thought of as *unsafe* either for itself or for elements of its environment [5, 180, 269]. While the potential danger, e.g., collisions, widely exist in the multi-robot tasks, the

difficulty of ascertaining safety is exacerbated in applying multi-agent RL (MARL). Due to the existence of multiple agents, MARL is faced with the difficulty of nonstationarity in training. Furthermore, for practical considerations, tackling safety in MARL is also challenging because each individual agent has to not only consider its own safety constraints, which already may conflict its reward maximisation, but also consider the safety constraints of others so that their joint behaviours are guaranteed to be safe. Currently, there are very few solutions that offer effective learning algorithms for safe multi-robot control problems. In this study, we address the problem with theoretical analysis, practical algorithms and three benchmarks.

The partial work of the study is shared on an open platform [89]. In this study, we provide a more comprehensive investigation and solution for safe MARL. Firstly, we investigate the safe multi-robot control problem from the safe MARL perspective. We develop both theoretical and practical algorithms that solve it. Secondly, we develop sophisticated environments for benchmarking safe MARL algorithms: Safe MAMuJoCo, Safe MARobosuite and Safe MAIG. Finally, we provide experiments to evaluate the effectiveness of our proposed methods. Besides, we also perform parameter studies to show sensitivity of the proposed algorithms to safety-related hyper-parameter changes. To the best of our knowledge, MACPO and MAPPO-Lagrangian are the first safety-aware model-free MARL algorithms and that work effectively in the challenging tasks with safety constraints.

5.2 Related Techniques of Safe Multi-Robot Reinforcement Learning

Robot learning safety is an important topic in the development of robotic systems. In safe reinforcement learning, [81], *Constrained Markov Decision Processes* (CMDPs) [10] are commonly employed to describe the safe control problem. At each step of decision making, the environment emits both rewards and costs of the decision, and agents need to maximise reward while avoid violating constraints on costs. Below, we review recent advances on safe RL for both single agent and multi-agent domains.

To tackle the learning problem in CMDPs, [5] introduced *Constrained Policy Optimisation* (CPO) which, like TRPO [216], updates the agent's policy under the trust region constraint to maximise a surrogate return, meanwhile obeying surrogate cost constraints. However, similar to TRPO, the computation of CPO may be expensive, because the Fisher information matrix still needs to be computed. An alternative solution is to apply primal-dual methods, giving rise to methods like TRPO-Lagrangian and PPO-Lagrangian [208]. Although these methods achieve impressive performance in terms of safety, their performance in terms of reward is poor [208]. Another class of algorithms that solves CMDPs is by [51, 52]; these algorithms leverage the theoretical property of the Lyapunov functions and propose safe value iteration and policy gradient procedures. In contrast to CPO, [51, 52] can work with off-policy methods; they also can be trained end-to-end with no need for line search that appears in CPO.

Safe multi-agent learning is an emerging research domain. Despite its importance [220], there are few sound solutions that work with MARL in a model-free setting. For example, CMIX [148] extends QMIX [206] by amending the reward function to take peak constraint violations into account, yet this approach does not come with safety guarantees during training. Furthermore, the majority of methods are designed specifically for learning robotics tasks. For example, the technique of barrier certificates [11, 34, 205] or model predictive shielding [289] from control theory are used to model safety. These methods, however, are derived from the traditional robotics point of view; unlike model-free MARL, they either are supervised learning based approaches, or require specific assumptions on the state space and environment dynamics. Moreover, due to the lack of a benchmark suite for safe MARL algorithms, the generalisation ability of those methods is unclear.

The most related work to ours is Safe Dec-PG [158], in which a primal-dual framework is adopted to find the saddle point between maximising reward and minimising cost. In particular, they proposed a decentralised policy gradient descent-ascent method through a consensus network. However, reaching a consensus equivalently imposes an extra constraint of parameter sharing among neighbouring agents, which could yield suboptimal solutions [131]. Furthermore, multi-agent policy gradient methods can suffer from high variance with growing state and action space sizes [132]. In contrast, our methods employ trust region optimisation and do not assume the parameter sharing.

HATRPO, introduced in [131], is the first multi-agent trust region method that enjoys theoretically-justified monotonic improvement guarantee. Its key idea is to make agents follow a sequential policy update scheme so that the expected joint advantage will always be positive, thus increasing reward. In this work, we show how to further develop this theory and derive a protocol which, in addition to the monotonic improvement, also guarantees to satisfy the safety constraints at every iteration during learning. The resulting algorithm (Algorithm 4) successfully attains theoretical guarantees of both monotonic improvement in reward and satisfaction of safety constraints.

5.3 Problem Formulation of Constrained Markov Game

We formulate the safe MARL problem as a *constrained Markov game* $\langle \mathcal{N}, \mathcal{S}, \mathcal{A}, p, \rho^0, \gamma, R, \mathcal{C}, \mathbf{c} \rangle$. Here, $\mathcal{N} = \{1, \dots, n\}$ is the set of agents, $\mathcal{S}$ is the state space, $\mathcal{A} = \prod_{i=1}^n \mathcal{A}^i$ is the product of the agents' action spaces, known as the joint action space, $p : \mathcal{S} \times \mathcal{A} \times \mathcal{S} \rightarrow \mathbb{R}$ is the probabilistic transition function, ρ^0 is the initial state distribution, $\gamma \in [0, 1)$ is the discount factor, $R : \mathcal{S} \times \mathcal{A} \rightarrow \mathbb{R}$ is the joint reward function, $\mathcal{C} = \{C_j^i\}_{1 \leq j \leq m^i}^{i \in \mathcal{N}}$ is the set of sets of cost functions (every agent i has m^i cost functions) of the form $C_j^i : \mathcal{S} \times \mathcal{A}^i \rightarrow \mathbb{R}$, and finally the set of corresponding cost-constraining values is given by $\mathbf{c} = \{c_j^i\}_{1 \leq j \leq m^i}^{i \in \mathcal{N}}$. At time step t , the agents are in a state s_t , and every agent i takes an action a_t^i according to its policy $\pi^i(a^i|s_t)$. Together with other agents' actions, it gives a joint action $\mathbf{a}_t = (a_t^1, \dots, a_t^n)$ and the joint policy $\pi(\mathbf{a}|s) = \prod_{i=1}^n \pi^i(a^i|s)$. The agents receive the reward $R(s_t, \mathbf{a}_t)$, meanwhile each agent i pays the costs $C_j^i(s_t, a_t^i)$, $\forall j = 1, \dots, m^i$. The environment then transits to a new state $s_{t+1} \sim p(\cdot|s_t, \mathbf{a}_t)$. In this paper, we consider a *fully-cooperative* setting where all agents share the same reward function, aiming to maximise the expected total reward of

$$J(\pi) \triangleq \mathbb{E}_{s_0 \sim \rho^0, \mathbf{a}_{0:\infty} \sim \pi, s_{1:\infty} \sim p} \left[\sum_{t=0}^{\infty} \gamma^t R(s_t, \mathbf{a}_t) \right],$$

meanwhile trying to satisfy every agent i 's safety constraints, written as

$$J_j^i(\pi) \triangleq \mathbb{E}_{s_0 \sim \rho^0, \mathbf{a}_{0:\infty} \sim \pi, s_{1:\infty} \sim p} \left[\sum_{t=0}^{\infty} \gamma^t C_j^i(s_t, a_t^i) \right] \leq c_j^i, \quad \forall j = 1, \dots, m^i. \quad (5.1)$$

We define the state-action value and the state-value functions in terms of reward as

$$Q_\pi(s, \mathbf{a}) \triangleq \mathbb{E}_{s_{1:\infty} \sim p, \mathbf{a}_{1:\infty} \sim \pi} \left[\sum_{t=0}^{\infty} \gamma^t R(s_t, \mathbf{a}_t) \middle| s_0 = s, \mathbf{a}_0 = \mathbf{a} \right],$$

$$V_\pi(s) \triangleq \mathbb{E}_{\mathbf{a} \sim \pi} [Q_\pi(s, \mathbf{a})].$$

The joint policies π that satisfy the Inequality (5.1) are referred to as **feasible**. Notably, in the above formulation, although the action a_t^i of agent i does not directly influence the costs

$\{C_j^k(s_t, a_t^k)\}_{j=1}^{m^k}$ of other agents $k \neq i$, the action a_t^i will implicitly influence their total costs due to the dependence on the next state s_{t+1} . We believe that this formulation realistically describes multi-agent interactions in the real-world; an action of an agent has an instantaneous effect on the system only locally, but the rest of agents may suffer from its consequences at later stages. For example, we consider a car that crosses on the red light: although other cars may not be at risk of riding into pedestrians immediately, the induced traffic may cause hazards soon later. For the j^{th} cost function of agent i , we define the j^{th} state-action cost value function and the state cost value function as

$$Q_{j,\pi}^i(s, a^i) \triangleq \mathbb{E}_{\mathbf{a}^{-i} \sim \pi^{-i}, s_{1:\infty} \sim p, \mathbf{a}_{1:\infty} \sim \pi} \left[\sum_{t=0}^{\infty} \gamma^t C_j^i(s_t, a_t^i) \mid s_0 = s, a_0^i = a^i \right],$$

$$V_{j,\pi}^i(s) \triangleq \mathbb{E}_{\mathbf{a} \sim \pi, s_{1:\infty} \sim p, \mathbf{a}_{1:\infty} \sim \pi} \left[\sum_{t=0}^{\infty} \gamma^t C_j^i(s_t, a_t^i) \mid s_0 = s \right].$$

Notably, the cost value functions $Q_{j,\pi}^i$ and $V_{j,\pi}^i$, although similar to traditional Q_π and V_π , involve extra indices i and j ; the superscript i denotes an agent, and the subscript j denotes its j^{th} cost.

Throughout this work, we pay close attention to the contribution to performance from different subsets of agents, therefore, we introduce the following notations. We denote an arbitrary subset $\{i_1, \dots, i_h\}$ of agents as $i_{1:h}$; we write $-i_{1:h}$ to refer to its complement. Given the agent subset $i_{1:h}$, we define the *multi-agent state-action value function*:

$$Q_\pi^{i_{1:h}}(s, \mathbf{a}^{i_{1:h}}) \triangleq \mathbb{E}_{\mathbf{a}^{-i_{1:h}} \sim \pi^{-i_{1:h}}} [Q_\pi(s, \mathbf{a}^{i_{1:h}}, \mathbf{a}^{-i_{1:h}})].$$

On top of it, for disjoint sets $j_{1:k}$ and $i_{1:h}^1$, the *multi-agent advantage function*² is defined as follows,

$$A_\pi^{i_{1:h}}(s, \mathbf{a}^{j_{1:k}}, \mathbf{a}^{i_{1:h}}) \triangleq Q_\pi^{j_{1:k}, i_{1:h}}(s, \mathbf{a}^{j_{1:k}}, \mathbf{a}^{i_{1:h}}) - Q_\pi^{j_{1:k}}(s, \mathbf{a}^{j_{1:k}}). \quad (5.2)$$

An interesting fact about the above multi-agent advantage function is that any advantage $A_\pi^{i_{1:h}}$ can be written as a sum of sequentially-unfolding multi-agent advantages of individual agents, and the advantage function is computed with GAE [217].

Lemma 9 (Multi-Agent Advantage Decomposition, [132]). *For any state $s \in \mathcal{S}$, subset of agents $i_{1:h} \subseteq \mathcal{N}$, and joint action $\mathbf{a}^{i_{1:h}}$, the following identity holds*

$$A_\pi^{i_{1:h}}(s, \mathbf{a}^{i_{1:h}}) = \sum_{j=1}^h A_\pi^{i_j}(s, \mathbf{a}^{i_{1:j-1}}, a^{i_j}).$$

Proof. We write the multi-agent advantage as in its definition, and expand it in a telescoping sum, where $A_\pi^{i_{1:h}}(s, \mathbf{a}^{i_{1:h}})$ is the advantage of agents $i_{1:h}$ taking $\mathbf{a}^{i_{1:h}}$.

$$\begin{aligned} A_\pi^{i_{1:h}}(s, \mathbf{a}^{i_{1:h}}) &= Q_\pi^{i_{1:h}}(s, \mathbf{a}^{i_{1:h}}) - V_\pi(s) \\ &= \sum_{j=1}^h [Q_\pi^{i_{1:j}}(s, \mathbf{a}^{i_{1:j}}) - Q_\pi^{i_{1:j-1}}(s, \mathbf{a}^{i_{1:j-1}})] \\ &= \sum_{j=1}^h A_\pi^{i_j}(s, \mathbf{a}^{i_{1:j-1}}, a^{i_j}). \end{aligned}$$

□

¹Where $i_{1:0}$ denotes the empty set, $A^{i_1}(s, a^{i_{1:0}}, a^{i_1}) = A^{i_1}(s, a^{i_1})$, $j_{1:k}, i_{1:h}$ denotes a union of two sets of agents.

²We would like to highlight that these multi-agent functions of $Q_\pi^{i_{1:h}}$ and $A_\pi^{i_{1:h}}$, although involve agents in superscripts, describe values *w.r.t* the reward rather than costs since they do not involve cost subscripts.

5.4 Multi-Agent Constrained Policy Optimisation

In this section, we first present a theoretically-justified safe multi-agent policy iteration procedure that leverages multi-agent trust region learning and constrained policy optimisation to solve constrained Markov games. Based on this, we propose two practical deep MARL algorithms, enabling optimising neural-network based policies that satisfy safety constraints. Throughout this work, we refer the symbols π and $\bar{\pi}$ to be the “current” and the “new” joint policies, respectively.

5.4.1 Multi-Agent Trust Region Learning With Constraints

[131] introduced the first multi-agent trust region method—HATRPO—that enjoys theoretically-justified monotonic improvement guarantee. Specifically, it relies on the multi-agent advantage decomposition in Lemma 9, and the “surrogate” return that is given as follows.

Definition 2. Let π be a joint policy, $\bar{\pi}^{i_{1:h-1}}$ be some other joint policy of agents $i_{1:h-1}$, and $\hat{\pi}^{i_h}$ be a policy of agent i_h . Then we define

$$L_{\pi}^{i_{1:h}}(\bar{\pi}^{i_{1:h-1}}, \hat{\pi}^{i_h}) \triangleq \mathbb{E}_{s \sim \rho_{\pi}, a^{i_{1:h-1}} \sim \bar{\pi}^{i_{1:h-1}}, a^{i_h} \sim \hat{\pi}^{i_h}} [A_{\pi}^{i_h}(s, a^{i_{1:h-1}}, a^{i_h})].$$

For any $\bar{\pi}^{i_{1:h-1}}$, we have

$$\begin{aligned} L_{\pi}^{i_{1:h}}(\bar{\pi}^{i_{1:h-1}}, \pi^{i_h}) &= \mathbb{E}_{s \sim \rho_{\pi}, a^{i_{1:h-1}} \sim \bar{\pi}^{i_{1:h-1}}, a^{i_h} \sim \pi^{i_h}} [A_{\pi}^{i_h}(s, a^{i_{1:h-1}}, a^{i_h})] \\ &= \mathbb{E}_{s \sim \rho_{\pi}, a^{i_{1:h-1}} \sim \bar{\pi}^{i_{1:h-1}}} [\mathbb{E}_{a^{i_h} \sim \pi^{i_h}} [A_{\pi}^{i_h}(s, a^{i_{1:h-1}}, a^{i_h})]] = 0. \end{aligned} \quad (5.3)$$

Building on Lemma 9 and Definition 2, we can observe,

$$L_{\pi}^{i_{1:h}}(\bar{\pi}^{i_{1:h-1}}, \pi^{i_h}) = \mathbb{E}_{s \sim \rho_{\pi}, a^{i_{1:h-1}} \sim \bar{\pi}^{i_{1:h-1}}, a^{i_h} \sim \pi^{i_h}} \left[\sum_{j=1}^h A_{\pi}^{i_j}(s, a^{i_{1:j-1}}, a^{i_j}) \right]. \quad (5.4)$$

With the above definition, we see that Lemma 9 allows for decomposing the joint surrogate return $L_{\pi}(\bar{\pi}) \triangleq \mathbb{E}_{s \sim \rho_{\pi}, a \sim \bar{\pi}} [A_{\pi}(s, a)]$ into a sum over surrogates of $L_{\pi}^{i_{1:h}}(\bar{\pi}^{i_{1:h-1}}, \pi^{i_h})$, for $h = 1, \dots, n$. This can be used to justify that if agents, with a joint policy π , update their policies by following a *sequential update scheme*, that is, if each agent in the subset $i_{1:h}$ sequentially solves the following optimisation problem:

$$\begin{aligned} \bar{\pi}^{i_h} &= \arg \max_{\hat{\pi}^{i_h}} L_{\pi}^{i_{1:h}}(\bar{\pi}^{i_{1:h-1}}, \hat{\pi}^{i_h}) - \nu D_{\text{KL}}^{\max}(\pi^{i_h}, \hat{\pi}^{i_h}), \\ \text{where } \nu &= \frac{4\gamma \max_{s,a} |A_{\pi}(s, a)|}{(1-\gamma)^2}, \\ \text{and } D_{\text{KL}}^{\max}(\pi^{i_h}, \hat{\pi}^{i_h}) &\triangleq \max_s D_{\text{KL}}(\pi^{i_h}(\cdot|s), \hat{\pi}^{i_h}(\cdot|s)), \end{aligned} \quad (5.5)$$

then the resulting joint policy $\bar{\pi}$ will surely improve the expected return, i.e., $J(\bar{\pi}) \geq J(\pi)$ (see the proof in [131, Lemma 2]). We know that due to the penalty term $D_{\text{KL}}^{\max}(\pi^{i_h}, \hat{\pi}^{i_h})$, the new policy $\bar{\pi}^{i_h}$ will stay close (w.r.t max-KL distance) to π^{i_h} .

For the safety constraints, we can extend Definition 2 to incorporate the “surrogate” cost, thus allowing us to study the cost functions in addition to the return.

Definition 3. Let π be a joint policy, and $\bar{\pi}^i$ be some other policy of agent i . Then, for any of its costs of index $j \in \{1, \dots, m^i\}$, we define

$$L_{j,\pi}^i(\bar{\pi}^i) = \mathbb{E}_{s \sim \rho_{\pi}, a^i \sim \bar{\pi}^i} [A_{j,\pi}^i(s, a^i)].$$

By generalising the result about the surrogate return in Equation (2), we can derive how the expected costs change when the agents update their policies. Specifically, we provide the following lemma.

Lemma 10. *Let π and $\bar{\pi}$ be joint policies. Let $i \in \mathcal{N}$ be an agent, and $j \in \{1, \dots, m^i\}$ be an index of one of its costs. The following inequality holds*

$$J_j^i(\bar{\pi}) \leq J_j^i(\pi) + L_{j,\pi}^i(\bar{\pi}^i) + \nu_j^i \sum_{h=1}^n D_{\text{KL}}^{\max}(\pi^h, \bar{\pi}^h),$$

where $\nu_j^i = \frac{4\gamma \max_{s,a^i} |A_{j,\pi}^i(s, a^i)|}{(1-\gamma)^2}$.

Proof. From the upper-bound version of Theorem 1 of [216] applied to joint policies π and $\bar{\pi}$, we conclude that

$$J_j^i(\bar{\pi}) \leq J_j^i(\pi) + \mathbb{E}_{s \sim \rho_\pi, a \sim \bar{\pi}} [A_{j,\pi}^i(s, a^i)] + \frac{4\alpha^2 \gamma \max_{s,a^i} |A_{j,\pi}^i(s, a^i)|}{(1-\gamma)^2},$$

where $\alpha = D_{\text{TV}}^{\max}(\pi, \bar{\pi}) = \max_s D_{\text{TV}}(\pi(\cdot|s), \bar{\pi}(\cdot|s))$.

Using Pinsker's $D_{\text{TV}}(p, q)^2 \leq D_{\text{KL}}(p, q)/2$ [135, 202, 216], we obtain

$$J_j^i(\bar{\pi}) \leq J_j^i(\pi) + \mathbb{E}_{s \sim \rho_\pi, a \sim \bar{\pi}} [A_{j,\pi}^i(s, a^i)] + \frac{2\gamma \max_{s,a^i} |A_{j,\pi}^i(s, a^i)|}{(1-\gamma)^2} D_{\text{KL}}^{\max}(\pi, \bar{\pi}),$$

where $D_{\text{KL}}^{\max}(\pi, \bar{\pi}) = \max_s D_{\text{KL}}(\pi(\cdot|s), \bar{\pi}(\cdot|s))$.

Notice now that we have $\mathbb{E}_{s \sim \rho_\pi, a \sim \bar{\pi}} [A_{j,\pi}^i(s, a^i)] = \mathbb{E}_{s \sim \rho_\pi, a^i \sim \bar{\pi}^i} [A_{j,\pi}^i(s, a^i)]$ as the action of agents other than i do not change the value of the variable inside of the expectation. Furthermore

$$\begin{aligned} D_{\text{KL}}^{\max}(\pi, \bar{\pi}) &= \max_s D_{\text{KL}}(\pi(\cdot|s), \bar{\pi}(\cdot|s)) = \max_s \left(\sum_{l=1}^n D_{\text{KL}}(\pi^l(\cdot|s), \bar{\pi}^l(\cdot|s)) \right) \\ &\leq \sum_{l=1}^n \max_s D_{\text{KL}}(\pi^l(\cdot|s), \bar{\pi}^l(\cdot|s)) = \sum_{l=1}^n D_{\text{KL}}^{\max}(\pi^l, \bar{\pi}^l). \end{aligned} \quad (5.6)$$

Setting $\nu_j^i = \frac{2\gamma \max_{s,a^i} |A_{j,\pi}^i(s, a^i)|}{(1-\gamma)^2}$, we finally obtain

$$J_j^i(\bar{\pi}) \leq J_j^i(\pi) + L_{j,\pi}^i(\bar{\pi}^i) + \nu_j^i \sum_{l=1}^n D_{\text{KL}}^{\max}(\pi^l, \bar{\pi}^l).$$

□

The above lemma suggests that, as long as the distances between the policies π^h and $\bar{\pi}^h$, $\forall h \in \mathcal{N}$, are sufficiently small, then the change in the j^{th} cost of agent i , i.e., $J_j^i(\bar{\pi}) - J_j^i(\pi)$, is controlled by the surrogate $L_{j,\pi}^i(\bar{\pi}^i)$. Importantly, this surrogate is independent of other agents' new policies. Hence, when the changes in policies of all agents are sufficiently small, each agent i can learn a better policy $\bar{\pi}^i$ by only considering its own surrogate return and surrogate costs. To summarise, we provide Algorithm 4 that guarantees both safety constraints satisfaction and monotonic performance improvement.

In this algorithm, in addition to sequentially maximising agents' surrogate returns, the agents must assure that their surrogate costs stay below the corresponding safety thresholds.

Algorithm 4 Safe Multi-Agent Policy Iteration with Monotonic Improvement Property

-
- 1: Initialise a joint policy $\pi_0 = (\pi_0^1, \dots, \pi_0^n)$. // initial unsafe policies are allowed, see experiments.
 - 2: **for** $k = 0, 1, \dots$ **do**
 - 3: Compute the advantage functions $A_{\pi_k}(s, \mathbf{a})$ and $A_{j, \pi_k}^i(s, a^i)$, for all state-(joint)action pairs $(s, \mathbf{a})$, agents i , and constraints $j \in \{1, \dots, m^i\}$.
 - 4: Compute $v = \frac{2\gamma \max_{s, \mathbf{a}} |A_{\pi_k}(s, \mathbf{a})|}{(1-\gamma)^2}$, and $v_j^i = \frac{2\gamma \max_{s, a^i} |A_{j, \pi_k}^i(s, a^i)|}{(1-\gamma)^2}$, $\forall i \in \mathcal{N}, j = 1, \dots, m^i$.
 - 5: Draw a permutation $i_{1:n}$ of agents arbitrarily.
 - 6: **for** $h = 1 : n$ **do**
 - 7: Compute the radius of the KL-constraint δ^{i_h} // see Equation (5.8) for the setup of δ^{i_h} .
 - 8: Make an update $\pi_{k+1}^{i_h} = \arg \max_{\pi^{i_h} \in \bar{\Pi}^{i_h}} [L_{\pi_k}^{i_{1:h}}(\pi_{k+1}^{i_{1:h-1}}, \pi^{i_h}) - v D_{\text{KL}}^{\max}(\pi_k^{i_h}, \pi^{i_h})]$,
 where $\bar{\Pi}^{i_h}$ is a subset of safe policies of agent i_h , given by

$$\begin{aligned} \bar{\Pi}^{i_h} = \{ & \pi^{i_h} \in \Pi^{i_h} \mid D_{\text{KL}}^{\max}(\pi_k^{i_h}, \pi^{i_h}) \leq \delta^{i_h}, \text{ and} \\ & J_j^{i_h}(\pi_k) + L_{j, \pi_k}^{i_h}(\pi^{i_h}) + v_j^{i_h} D_{\text{KL}}^{\max}(\pi_k^{i_h}, \pi^{i_h}) \leq \\ & c_j^{i_h} - \sum_{l=1}^{h-1} v_j^{i_l} D_{\text{KL}}^{\max}(\pi_k^{i_l}, \pi^{i_l}), \forall j = 1, \dots, m^{i_h} \}. \end{aligned} \quad (5.7)$$
 - 9: **end for**
 - 10: **end for**
-

Meanwhile, they have to constrain their policy search to small local neighbourhoods (w.r.t max-KL distance). As such, Algorithm 4 demonstrates two desirable properties: reward performance improvement and satisfaction of safety constraints, as stated by Theorem 6.

In Algorithm 4, we compute the size of KL constraint as

$$\delta^{i_h} = \min \left\{ \min_{l \leq h-1} \min_{1 \leq j \leq m^l} \frac{c_j^{i_l} - J_j^{i_l}(\pi_k) - L_{j, \pi_k}^{i_l}(\pi_{k+1}^{i_l}) - v_j^{i_l} \sum_{u=1}^{h-1} D_{\text{KL}}^{\max}(\pi_k^u, \pi_{k+1}^u)}{v_j^{i_l}}, \right. \\ \left. \min_{l \geq h+1} \min_{1 \leq j \leq m^l} \frac{c_j^{i_l} - J_j^{i_l}(\pi_k) - v_j^{i_l} \sum_{u=1}^{h-1} D_{\text{KL}}^{\max}(\pi_k^u, \pi_{k+1}^u)}{v_j^{i_l}} \right\}. \quad (5.8)$$

Note that δ^{i_1} (i.e., $h = 1$) is guaranteed to be non-negative if π_k satisfies safety constraints; that is because then $c_j^{i_l} \geq J_j^{i_l}(\pi_k)$ for all l and j , and the set $\{l \mid l < h\}$ is empty.

This formula for δ^{i_h} , combined with Lemma 10, assures that the policies π^{i_h} within δ^{i_h} max-KL distance from $\pi_k^{i_h}$ will not violate other agents' safety constraints, as long as the base joint policy π_k did not violate them (which assures $\delta^{i_1} \geq 0$). To see this, notice that for every $l = 1, \dots, h-1$, and $j = 1, \dots, m^l$,

$$D_{\text{KL}}^{\max}(\pi_k^{i_h}, \pi^{i_h}) \leq \delta^{i_h} \leq \frac{c_j^{i_l} - J_j^{i_l}(\pi_k) - L_{j, \pi_k}^{i_l}(\pi_{k+1}^{i_l}) - v_j^{i_l} \sum_{u=1}^{h-1} D_{\text{KL}}^{\max}(\pi_k^u, \pi_{k+1}^u)}{v_j^{i_l}},$$

$$\text{implies } J_j^{i_l}(\pi_k) + L_{j, \pi_k}^{i_l}(\pi_{k+1}^{i_l}) + v_j^{i_l} \sum_{u=1}^{h-1} D_{\text{KL}}^{\max}(\pi_k^u, \pi_{k+1}^u) + v_j^{i_l} D_{\text{KL}}^{\max}(\pi_k^{i_h}, \pi^{i_h}) \leq c_j^{i_l}.$$

By Lemma 10, the left-hand side of the above inequality is an upper bound of $J_j^{i_l}(\pi_{k+1}^{i_{1:h-1}}, \pi^{i_h}, \pi_k^{-i_{1:h}})$, which implies that the update of agent i_h does not violate the constraint of $J_j^{i_l}$. The fact that

the constraints of $J_j^{i_l}$ for $l \geq h+1$ are not violated, i.e.,

$$J_j^{i_l}(\pi_k) + \nu_j^{i_l} \sum_{u=1}^{h-1} D_{\text{KL}}^{\max}(\pi_k^u, \pi_{k+1}^u) + \nu_j^{i_l} D_{\text{KL}}^{\max}(\pi_k^{i_h}, \pi_{k+1}^{i_h}) \leq c_j^{i_l}, \quad (5.9)$$

is analogous.

Theorem 6. *If a sequence of joint policies $(\pi_k)_{k=0}^{\infty}$ is obtained from Algorithm 4, then it has the monotonic improvement property, $J(\pi_{k+1}) \geq J(\pi_k)$, as well as it satisfies the safety constraints, $J_j^i(\pi_k) \leq c_j^i$, for all $k \in \mathbb{N}$, $i \in \mathcal{N}$, and $j \in \{1, \dots, m^i\}$.*

Proof. Safety constraints are assured to be met if the agents' update size is bounded by Equation (5.8). It suffices to show the monotonic improvement property. By Theorem 1 from [216], we have

$$\begin{aligned} J(\pi_{k+1}) &\geq J(\pi_k) + \mathbb{E}_{s \sim \rho_{\pi_k}, \mathbf{a} \sim \pi_{k+1}} [A_{\pi_k}(s, \mathbf{a})] - \nu D_{\text{KL}}^{\max}(\pi_k, \pi_{k+1}), \\ &\text{which by Equation 5.6 is lower-bounded by} \\ &\geq J(\pi_k) + \mathbb{E}_{s \sim \rho_{\pi_k}, \mathbf{a} \sim \pi_{k+1}} [A_{\pi_k}(s, \mathbf{a})] - \sum_{h=1}^n \nu D_{\text{KL}}^{\max}(\pi_k^{i_h}, \pi_{k+1}^{i_h}) \\ &\text{which by Lemma 9 equals} \\ &= J(\pi_k) + \sum_{h=1}^n \mathbb{E}_{s \sim \rho_{\pi_k}, \mathbf{a}^{1:h} \sim \pi_{k+1}^{1:h}} [A_{\pi_k}^{i_h}(s, \mathbf{a}^{1:h-1}, a^{i_h})] - \sum_{h=1}^n \nu D_{\text{KL}}^{\max}(\pi_k^{i_h}, \pi_{k+1}^{i_h}) \\ &= J(\pi_k) + \sum_{h=1}^n (L_{\pi_k}^{i_h}(\pi_{k+1}^{1:h-1}, \pi_{k+1}^{i_h}) - \nu D_{\text{KL}}^{\max}(\pi_k^{i_h}, \pi_{k+1}^{i_h})), \end{aligned} \quad (5.10)$$

and as for every h , $\pi_{k+1}^{i_h}$ is the argmax, this is lower-bounded by

$$\geq J(\pi_k) + \sum_{h=1}^n (L_{\pi_k}^{i_h}(\pi_{k+1}^{1:h-1}, \pi_k^{i_h}) - \nu D_{\text{KL}}^{\max}(\pi_k^{i_h}, \pi_k^{i_h})),$$

which, as follows from Definition 2 and Equation (5.3), equals

$$= J(\pi_k) + \sum_{h=1}^n 0 = J(\pi_k), \text{ which finishes the proof.}$$

□

Theorem 6 assures that agents following Algorithm 4 will not only explore safe policies; meanwhile, every new policy will be guaranteed to result in performance improvement. These two properties hold under the condition that only restrictive policy updates are made; this is due to the KL-penalty term in every agent's objective (i.e., $\nu D_{\text{KL}}^{\max}(\pi_k^{i_h}, \pi_{k+1}^{i_h})$), as well as the constraints on cost surrogates (i.e., the conditions in $\bar{\Pi}^{i_h}$). In practice, it can be intractable to evaluate $D_{\text{KL}}(\pi_k^{i_h}(\cdot|s), \pi_{k+1}^{i_h}(\cdot|s))$ at every state in order to compute $D_{\text{KL}}^{\max}(\pi_k^{i_h}, \pi_{k+1}^{i_h})$. In the following subsections, we describe how to approximate Algorithm 4 in the case of parameterised policies.

5.4.2 MACPO: Multi-Agent Constrained Policy Optimisation

In this section, we focus on the practical implementation where large state and action spaces prevent agents from designating policies $\pi^i(\cdot|s)$ for each state separately. To handle this, we parameterise each agent's policy $\pi_{\theta^i}^i$ by a neural network with parameter θ^i . Correspondingly, the joint policies π_{θ} are parametrised by $\theta = (\theta^1, \dots, \theta^n)$.

At each iteration of Algorithm 4, every agent i_h maximises its surrogate return with a KL-penalty, subject to surrogate cost constraint. Yet, direct computation of the max-KL constraint is intractable in practical settings, as it would require computation of KL-divergence at every single state. Instead, we relax it by adopting a form of expected KL-constraint

$$\overline{D}_{\text{KL}}(\pi_k^{i_h}, \pi^{i_h}) \leq \delta,$$

where $\overline{D}_{\text{KL}}(\pi_k^{i_h}, \pi^{i_h}) \triangleq \mathbb{E}_{s \sim \rho_{\pi_k}} [D_{\text{KL}}(\pi_k^{i_h}(\cdot|s), \pi^{i_h}(\cdot|s))]$.

We approximate these expectations by sampling. As a result, the optimisation problem solved by agent i_h can be written as

$$\begin{aligned} \theta_{k+1}^{i_h} &= \arg \max_{\theta^{i_h}} \mathbb{E}_{s \sim \rho_{\pi_{\theta_k}}, \mathbf{a}^{1:h-1} \sim \pi_{\theta_k^{i_h}}^{1:h-1}, \mathbf{a}^{i_h} \sim \pi_{\theta_{k+1}^{i_h}}^{i_h}} \left[A_{\pi_{\theta_k}}^{i_h}(s, \mathbf{a}^{1:h-1}, \mathbf{a}^{i_h}) \right] \\ \text{s.t. } & J_j^{i_h}(\pi_{\theta_k}) + \mathbb{E}_{s \sim \rho_{\pi_{\theta_k}}, \mathbf{a}^{i_h} \sim \pi_{\theta_k^{i_h}}^{i_h}} \left[A_{j, \pi_{\theta_k}}^{i_h}(s, \mathbf{a}^{i_h}) \right] \leq c_j^{i_h}, \quad \forall j \in \{1, \dots, m^{i_h}\}, \\ & \text{and } \overline{D}_{\text{KL}}(\pi_k^{i_h}, \pi^{i_h}) \leq \delta. \end{aligned} \quad (5.11)$$

To solve this problem, we approximate Equation (5.11) with the Taylor expansion of the optimisation objective and cost constraints up to the first order, and the KL-divergence up to the second order. Consequently, the optimisation problem (5.11) can be written as

$$\begin{aligned} \theta_{k+1}^{i_h} &= \arg \max_{\theta^{i_h}} (\mathbf{g}^{i_h})^T (\theta^{i_h} - \theta_k^{i_h}) \\ \text{s.t. } & d_j^{i_h} + (\mathbf{b}_j^{i_h})^T (\theta^{i_h} - \theta_k^{i_h}) \leq 0, \quad j = 1, \dots, m \\ & \frac{1}{2} (\theta^{i_h} - \theta_k^{i_h})^T \mathbf{H}^{i_h} (\theta^{i_h} - \theta_k^{i_h}) \leq \delta, \end{aligned} \quad (5.12)$$

where $\mathbf{g}^{i_h}$ is the gradient of the objective of agent i_h in Equation (5.11), $d_j^i = J_j^i(\pi_{\theta_k}) - c_j^i$ and $\mathbf{H}^{i_h} = \nabla_{\theta^{i_h}}^2 \overline{D}_{\text{KL}}(\pi_{\theta_k^{i_h}}^{i_h}, \pi^{i_h})|_{\theta^{i_h} = \theta_k^{i_h}}$, is the Hessian of the average KL divergence of agent i_h , and $\mathbf{b}_j^{i_h}$ is the gradient of agent of the j^{th} constraint of agent i_h .

Similar to [5] and [50], we can take a primal-dual optimisation approach to solve the linear quadratic optimisation in Equation (5.12). Specifically, the dual form is given by

$$\max_{\lambda^{i_h} \geq 0, \mathbf{v}^{i_h} \geq 0} \frac{-1}{2\lambda^{i_h}} [(\mathbf{g}^{i_h})^T (\mathbf{H}^{i_h})^{-1} \mathbf{g}^{i_h} - 2(\mathbf{r}^{i_h})^T \mathbf{v}^{i_h} + (\mathbf{v}^{i_h})^T \mathbf{S}^{i_h}] + (\mathbf{v}^{i_h})^T \mathbf{c}^{i_h} - \frac{\lambda^{i_h} \delta}{2}, \quad (5.13)$$

where $\mathbf{B}^{i_h} = [\mathbf{b}_1^{i_h}, \dots, \mathbf{b}_m^{i_h}]$, $\mathbf{r}^{i_h} \triangleq (\mathbf{g}^{i_h})^T (\mathbf{H}^{i_h})^{-1} \mathbf{B}^{i_h}$, $\mathbf{S}^{i_h} \triangleq (\mathbf{B}^{i_h})^T (\mathbf{H}^{i_h})^{-1} \mathbf{B}^{i_h}$.

Given the solution to the dual form in Equation (5.13), i.e., $\lambda_*^{i_h}$ and $\mathbf{v}_*^{i_h}$, the solution to the primal problem in Equation (5.12) can thus be written by

$$\theta_*^{i_h} = \theta_k^{i_h} + \frac{1}{\lambda_*^{i_h}} (\mathbf{H}^{i_h})^{-1} (\mathbf{g}^{i_h} - \mathbf{B}^{i_h} \mathbf{v}_*^{i_h}).$$

See 5.8 for derivation. In practice, we use backtracking line search starting at $1/\lambda_*^{i_h}$ to choose the step size of the above update. Furthermore, we note that the optimisation step in Equation (5.12) is an approximation to the original problem from Equation (5.11); therefore, it is possible that an infeasible policy $\pi_{\theta_{k+1}^{i_h}}^{i_h}$ will be generated. Fortunately, as the policy optimisation takes place in the trust region of $\pi_{\theta_k^{i_h}}^{i_h}$, the size of update is small, and a feasible

policy can be easily recovered. We do so by means of a negative TRPO step applied on the cost of agent i_h :

$$\theta_{k+1}^{i_h} = \theta_k^{i_h} - \alpha^j \sqrt{\frac{2\delta}{\mathbf{b}^{i_h T} (\mathbf{H}^{i_h})^{-1} \mathbf{b}^{i_h}}} (\mathbf{H}^{i_h})^{-1} \mathbf{b}^{i_h}. \quad (5.14)$$

To put it together, we refer to this algorithm as *MACPO*, and provide its pseudocode in Algorithm 5. The computational complexity of MACPO is $\mathcal{O}(KMHP^3)$, where K denotes the number of steps, M denotes the number of agents, H denotes the number of constraints, P denotes the number of policy parameters.

Algorithm 5 MACPO

- 1: **Input:** Stepsize α , batch size B , number of: agents n , episodes K , steps per episode T , possible steps in line search L .
 - 2: **Initialize:** Actor networks $\{\theta_0^i, \forall i \in \mathcal{N}\}$, V-value network $\{\phi_0\}$, V^i -cost networks $\{\phi_{j,0}^i\}_{i=1:n, j=1:m}$, Replay buffer $\mathcal{B}$.
 - 3: **for** $k = 0, 1, \dots, K-1$ **do**
 - 4: Collect a set of trajectories by running the joint policy π_{θ_k} .
 - 5: Push transitions $\{(o_t^i, a_t^i, o_{t+1}^i, r_t), \forall i \in \mathcal{N}, t \in T\}$ into $\mathcal{B}$.
 - 6: Sample a random minibatch of M transitions from $\mathcal{B}$.
 - 7: Compute advantage function $\hat{A}(s, \mathbf{a})$ based on V-value network with GAE.
 - 8: Compute cost-advantage functions $\hat{A}_j^i(s, a^i)$ based on V^i -cost critics with GAE.
 - 9: Draw a random permutation of agents $i_{1:n}$.
 - 10: Set $M^{i_1}(s, \mathbf{a}) = \hat{A}(s, \mathbf{a})$.
 - 11: **for** agent $i_h = i_1, \dots, i_n$ **do**
 - 12: Estimate the gradient of the agent's maximisation objective (5.15).
 - 13: **for** $j = 1, \dots, m^{i_h}$ **do**
 - 14: Estimate the gradient of the agent's j^{th} cost (5.16).
 - 15: **end for**
 - 16: Set $\hat{\mathbf{B}}^{i_h} = [\hat{\mathbf{b}}_1^{i_h}, \dots, \hat{\mathbf{b}}_m^{i_h}]$.
 - 17: Compute $\hat{\mathbf{H}}_k^{i_h}$, the Hessian of the average KL-divergence
$$\frac{1}{BT} \sum_{b=1}^B \sum_{t=1}^T D_{\text{KL}} \left(\pi_{\theta_k^{i_h}}^{i_h}(\cdot | o_t^{i_h}), \pi_{\theta^{i_h}}^{i_h}(\cdot | o_t^{i_h}) \right).$$
 - 18: **if** the problem is feasible **then**
 - 19: Solve the dual (5.13) for $\lambda_*^{i_h}, \mathbf{v}_*^{i_h}$.
 - 20: Use the conjugate gradient algorithm to compute the update direction (5.17).
 - 21: Update agent i_h 's policy by (5.18), where $j \in \{0, 1, \dots, L\}$ is the smallest such j which improves the sample loss, and satisfies the sample constraints, found by the backtracking line search.
 - 22: **else**
 - 23: Use equation (5.14) to recover policy $\theta_{k+1}^{i_h}$ from unfeasible points.
 - 24: **end if**
 - 25: Compute (5.19). //Unless $h = n$.
 - 26: **end for**
 - 27: Update V-value network by formula (5.20).
 - 28: **end for**
-

$$\hat{\mathbf{g}}_k^{i_h} = \frac{1}{B} \sum_{b=1}^B \sum_{t=1}^T \nabla_{\theta_k^{i_h}} \log \pi_{\theta_k^{i_h}}^{i_h} (a_t^{i_h} | o_t^{i_h}) M^{i_{1:h}}(s_t, \mathbf{a}_t), \quad (5.15)$$

$$\hat{\mathbf{b}}_j^{i_h} = \frac{1}{B} \sum_{b=1}^B \sum_{t=1}^T \nabla_{\theta_k^{i_h}} \log \pi_{\theta_k^{i_h}}^{i_h} (a_t^{i_h} | o_t^{i_h}) \hat{A}_j^{i_h}(s_t, a_t^{i_h}), \quad (5.16)$$

$$\mathbf{x}_k^{i_h} = (\hat{\mathbf{H}}_k^{i_h})^{-1} (\mathbf{g}_k^{i_h} - \hat{\mathbf{B}}^{i_h} \mathbf{v}_*^{i_h}), \quad (5.17)$$

$$\theta_{k+1}^{i_h} = \theta_k^{i_h} + \frac{\alpha^j}{\lambda_*^{i_h}} \hat{\mathbf{x}}_k^{i_h}, \quad (5.18)$$

$$M^{i_{1:h+1}}(s, \mathbf{a}) = \frac{\pi_{\theta_{k+1}^{i_h}}^{i_h} (a^{i_h} | o^{i_h})}{\pi_{\theta_k^{i_h}}^{i_h} (a^{i_h} | o^{i_h})} M^{i_{1:h}}(s_t, \mathbf{a}_t), \quad (5.19)$$

$$\phi_{k+1} = \arg \min_{\phi} \frac{1}{\mathcal{N}} \frac{1}{T} \sum_{n=1}^{\mathcal{N}} \sum_{t=0}^T (V_{\phi}(s_t) - \hat{R}_t)^2. \quad (5.20)$$

5.4.3 MAPPO-Lagrangian

As an alternative to MACPO, one can use Lagrangian multipliers in place of optimisation with linear and quadratic constraints to solve Equation (5.11). The Lagrangian method is simple to implement, and it does not require the repetitive computation of the Hessian $\mathbf{H}^{i_h}$ whose size grows quadratically with the dimension of the parameter vector θ^{i_h} .

Before we proceed, let us briefly recall the optimisation procedure with a Lagrangian multiplier. Suppose that our goal is to maximise a bounded real-valued function $f(x)$ under a constraint $g(x)$; $\max_x f(x)$, s.t. $g(x) \leq 0$. We can introduce a scalar variable λ and reformulate the optimisation by

$$\max_x \min_{\lambda \geq 0} f(x) - \lambda g(x). \quad (5.21)$$

Suppose that x_+ satisfies $g(x_+) > 0$. This immediately implies that $-\lambda g(x_+) \rightarrow -\infty$, as $\lambda \rightarrow +\infty$, and so Equation (5.21) equals $-\infty$ for $x = x_+$. On the other hand, if x_- satisfies $g(x_-) < 0$, we have that $-\lambda g(x_-) \geq 0$, with equality only for $\lambda = 0$. In that case, the optimisation objective's value equals $f(x_-) > -\infty$. Hence, the only candidate solutions to the problem are those x that satisfy the constraint $g(x) \leq 0$, and the objective matches with $f(x)$.

We can employ the above trick to the constrained optimisation problem from Equation (5.11) by subsuming the cost constraints into the optimisation objective with Lagrangian multipliers. As such, agent i_h computes $\bar{\lambda}_{1:m^{i_h}}^{i_h}$ and $\theta_{k+1}^{i_h}$ to solve the following min-max

optimisation problem

$$\begin{aligned} \max_{\theta^{i_h}} \min_{\lambda_{1:m^{i_h}} \geq 0} & \left[\mathbb{E}_{s \sim \rho, \pi_{\theta_k}, \mathbf{a}^{i_{1:h-1}} \sim \pi_{\theta_{k+1}}^{i_{1:h-1}}, \mathbf{a}^{i_h} \sim \pi_{\theta^{i_h}}^{i_h}} \left[A_{\pi_{\theta_k}}^{i_h}(s, \mathbf{a}^{i_{1:h-1}}, \mathbf{a}^{i_h}) \right] \right. \\ & \left. - \sum_{u=1}^{m^{i_h}} \lambda_u^{i_h} \left(\mathbb{E}_{s \sim \rho, \pi_{\theta_k}, \mathbf{a}^{i_h} \sim \pi_{\theta^{i_h}}^{i_h}} \left[A_{u, \pi_{\theta_k}}^{i_h}(s, \mathbf{a}^{i_h}) \right] + d_u^{i_h} \right) \right], \\ \text{s.t. } & \bar{D}_{\text{KL}}(\pi_{\theta_k}^{i_h}, \pi_{\theta^{i_h}}^{i_h}) \leq \delta. \end{aligned} \quad (5.22)$$

Although the objective from Equation (5.22) is affine in the Lagrangian multipliers $\lambda_u^{i_h}$ ($u = 1, \dots, m^{i_h}$), which enables gradient-based optimisation, computing the KL-divergence constraint still complicates the overall process. To handle this, one can further simplify it by adopting the *PPO-clip* objective [218], which replaces the KL-divergence constraint with the *clip* operator, and updates the policy parameter with first-order methods. We do so by defining

$$A_{\pi_{\theta_k}}^{i_h, (\lambda)}(s, \mathbf{a}^{i_{1:h-1}}, \mathbf{a}^{i_h}) \triangleq A_{\pi_{\theta_k}}^{i_h}(s, \mathbf{a}^{i_{1:h-1}}, \mathbf{a}^{i_h}) - \sum_{u=1}^{m^{i_h}} \lambda_u^{i_h} \left(A_{u, \pi_{\theta_k}}^{i_h}(s, \mathbf{a}^{i_h}) + d_u^{i_h} \right),$$

and rewriting the Equation (5.22) as

$$\begin{aligned} \max_{\theta^{i_h}} \min_{\lambda_{1:m^{i_h}} \geq 0} & \mathbb{E}_{s \sim \rho, \pi_{\theta_k}, \mathbf{a}^{i_{1:h-1}} \sim \pi_{\theta_{k+1}}^{i_{1:h-1}}, \mathbf{a}^{i_h} \sim \pi_{\theta^{i_h}}^{i_h}} \left[A_{\pi_{\theta_k}}^{i_h, (\lambda)}(s, \mathbf{a}^{i_{1:h-1}}, \mathbf{a}^{i_h}) \right], \\ \text{s.t. } & \bar{D}_{\text{KL}}(\pi_{\theta_k}^{i_h}, \pi_{\theta^{i_h}}^{i_h}) \leq \delta. \end{aligned} \quad (5.23)$$

The objective in Equation (5.23) takes a form of an expectation with a KL-divergence constraint on the policy. To approximately adjust the distance between old and new policies, the *clip* operator is leveraged. Finally, the objective takes the form

$$\begin{aligned} \mathbb{E}_{s \sim \rho, \pi_{\theta_k}, \mathbf{a}^{i_{1:h-1}} \sim \pi_{\theta_{k+1}}^{i_{1:h-1}}, \mathbf{a}^{i_h} \sim \pi_{\theta^{i_h}}^{i_h}} & \left[\min \left(\frac{\pi_{\theta^{i_h}}^{i_h}(\mathbf{a}^{i_h} | s)}{\pi_{\theta_k}^{i_h}(\mathbf{a}^{i_h} | s)} A_{\pi_{\theta_k}}^{i_h, (\lambda)}(s, \mathbf{a}^{i_{1:h-1}}, \mathbf{a}^{i_h}), \right. \right. \\ & \left. \left. \text{clip} \left(\frac{\pi_{\theta^{i_h}}^{i_h}(\mathbf{a}^{i_h} | s)}{\pi_{\theta_k}^{i_h}(\mathbf{a}^{i_h} | s)}, 1 \pm \epsilon \right) A_{\pi_{\theta_k}}^{i_h, (\lambda)}(s, \mathbf{a}^{i_{1:h-1}}, \mathbf{a}^{i_h}) \right) \right]. \end{aligned} \quad (5.24)$$

The clip operator replaces the policy ratio with $1 - \epsilon$, or $1 + \epsilon$, depending on whether its value is below or above the threshold interval. As such, agent i_h can learn within its trust region by updating θ^{i_h} to maximise Equation (5.24), while the Lagrangian multipliers are updated towards the direction opposite to their gradients of Equation (5.22), which can be computed analytically. We name this algorithm *MAPPO-Lagrangian*, and give its pseudocode in Algorithm 6. The computational complexity of MAPPO-L is $\mathcal{O}(KMNHP)$, where K denotes the number of steps, M denotes the number of agents, N denotes the number of PPO-Epoch, H denotes the number of constraints, P denotes the number of policy parameters.

$$M^{i_h, (\lambda)}(s_t, \mathbf{a}_t) = M^{i_h}(s_t, \mathbf{a}_t) - \sum_{j=1}^n \lambda_j^{i_h} \hat{A}_j^{i_h}(s_t, \mathbf{a}_t), \quad (5.25)$$

Algorithm 6 MAPPO-Lagrangian

```

1: Input: Stepsizes  $\alpha_\theta, \alpha_\lambda$ , batch size  $B$ , number of: agents  $n$ , episodes  $K$ , steps per episode
    $T$ , discount factor  $\gamma$ .
2: Initialize: Actor networks  $\{\theta_0^i, \forall i \in \mathcal{N}\}$ , V-value network  $\{\phi_0\}$ ,
   V-cost networks  $\{\phi_{j,0}^i\}_{1 \leq j \leq m^i}^{i \in \mathcal{N}}$ , Replay buffer  $\mathcal{B}$ .
3: for  $k = 0, 1, \dots, K-1$  do
4:   Collect a set of trajectories by running the joint policy  $\pi_{\theta_k}$ .
5:   Push transitions  $\{(o_t^i, a_t^i, o_{t+1}^i, r_t), \forall i \in \mathcal{N}, t \in T\}$  into  $\mathcal{B}$ .
6:   Sample a random minibatch of  $B$  transitions from  $\mathcal{B}$ .
7:   Compute advantage function  $\hat{A}(s, \mathbf{a})$  based on V-value network with GAE.
8:   Compute cost advantage functions  $\hat{A}_j^i(s, \mathbf{a}^i)$  for all agents and costs,
      based on V-cost networks with GAE.
9:   Draw a random permutation of agents  $i_{1:n}$ .
10:  Set  $M^{i_1}(s, \mathbf{a}) = \hat{A}(s, \mathbf{a})$ .
11:  for agent  $i_h = i_1, \dots, i_n$  do
12:    Initialise a policy parameter  $\theta_k^{i_h} = \theta_k^{i_h}$ ,
      and Lagrangian multipliers  $\lambda_j^{i_h} = 0, \forall j = 1, \dots, m^{i_h}$ .
13:    Compute the Lagrangian in Equation (5.25).
14:    for  $e = 1, \dots, e_{\text{PPO}}$  do
15:      Compute the gradient  $\Delta_{\theta^{i_h}}$  of the Lagrangian PPO-Clip objective with Equation
        (5.26).
16:      Update temporarily the actor paramaters
          
$$\theta^{i_h} \leftarrow \theta^{i_h} + \alpha_\theta \Delta_{\theta^{i_h}}.$$

17:      for  $j = 1, \dots, m^{i_h}$  do
18:        Approximate the constraint violation via Equation (5.27).
19:        Differentiate the constraint via Equation (5.28).
20:      end for
21:      for  $j = 1, \dots, m^{i_h}$  do
22:        Update temporarily the Lagrangian multiplier as in Equation (5.29).
23:      end for
24:    end for
25:    Update the actor parameter  $\theta_{k+1}^{i_h} = \theta^{i_h}$ .
26:    Compute Equation (5.30). //Unless  $h = n$ .
27:  end for
28:  Update the critic via Equation (5.31).
29: end for

```

$$\Delta_{\theta^{i_h}} = \nabla_{\theta^{i_h}} \frac{1}{B} \sum_{b=1}^B \sum_{t=0}^T \min \left(\frac{\pi_{\theta^{i_h}}^{i_h}(a_t^{i_h} | o_t^{i_h})}{\pi_{\theta_k^{i_h}}^{i_h}(a_t^{i_h} | o_t^{i_h})} M^{i_h,(\lambda)}(s_t, \mathbf{a}_t), \right. \\ \left. \text{clip} \left(\frac{\pi_{\theta^{i_h}}^{i_h}(a_t^{i_h} | o_t^{i_h})}{\pi_{\theta_k^{i_h}}^{i_h}(a_t^{i_h} | o_t^{i_h})}, 1 \pm \epsilon \right) M^{i_h,(\lambda)}(s_t, \mathbf{a}_t) \right), \quad (5.26)$$

$$d_j^{i_h} = \frac{1}{BT} \sum_{b=1}^B \sum_{t=1}^T \hat{V}_j^{i_h}(s_t) - c_j^{i_h}, \quad (5.27)$$

$$\Delta \lambda_j^{i_h} = \frac{-1}{B} \sum_{b=1}^B \left(d_j^{i_h} (1 - \gamma) + \sum_{t=0}^T \frac{\pi_{\theta^{i_h}}^{i_h}(a_t^{i_h} | o_t^{i_h})}{\pi_{\theta_k^{i_h}}^{i_h}(a_t^{i_h} | o_t^{i_h})} \hat{A}_j^{i_h}(s_t, a_t^{i_h}) \right), \quad (5.28)$$

$$\lambda_j^{i_h} \leftarrow \left(\lambda_j^{i_h} - \alpha_\lambda \Delta \lambda_j^{i_h} \right)_+, \quad (5.29)$$

$$M^{i_{h+1}}(s, \mathbf{a}) = \frac{\pi_{\theta_{k+1}^{i_h}}^{i_h}(a^{i_h} | o^{i_h})}{\pi_{\theta_k^{i_h}}^{i_h}(a^{i_h} | o^{i_h})} M^{i_h}(s, \mathbf{a}), \quad (5.30)$$

$$\phi_{k+1} = \arg \min_{\phi} \frac{1}{BT} \sum_{b=1}^B \sum_{t=0}^T (V_{\phi}(s_t) - \hat{R}_t)^2. \quad (5.31)$$

5.5 Experiments of Safe Multi-Robot Learning

In this section, first, we introduce three safe MARL benchmarks. Second, we provide and analyse results of experiments conducted in these environments. Lastly, for completeness, we give the details of settings for the experiments at the end of the section.

5.5.1 Safe Multi-Agent MuJoCo Tasks

Although MARL researchers have long had a variety of environments to test different algorithms, such as StarCraftII [212] and Multi-Agent MuJoCo [195], no public safe MARL benchmark has been proposed; this impedes researchers from evaluating and benchmarking safety-aware multi-agent learning methods. As a key contribution of this paper, we introduce three Safe MARL benchmarks, Safe MAMuJoCo, Safe MARobosuite and Safe MAIG. We use these environments to evaluate the performance of our methods, in terms of reward and safety, against strong baselines.

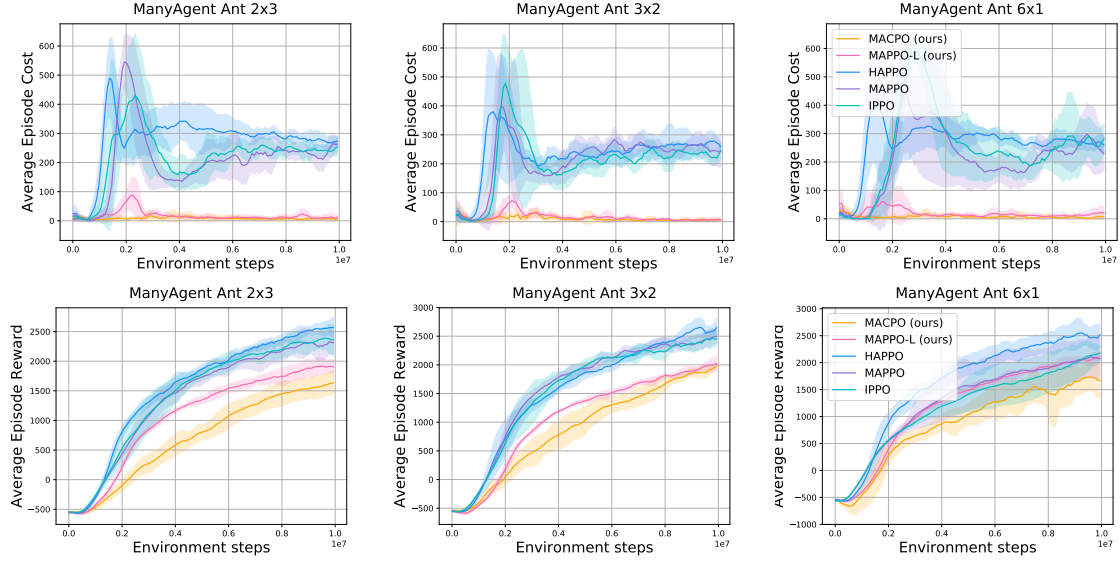


Figure 5.1: Performance comparisons on Safe ManyAgent Ant task 1.0 in terms of cost and reward. The safety constraint value is 1. Each subfigure represents a different robot, within each subfigure, three task setups in terms of multi-agent control are considered, and for each task we plot the cost curves (the lower the better) in the upper row and the reward curves (the higher the better) in the bottom row.

In this section, we use Safe MAMuJoCo to examine if the MACPO/MAPPO-Lagrangian agents can satisfy their safety constraints and cooperatively learn to achieve high rewards, compared to existing MARL algorithms. Notably, our proposed methods adopt two different approaches for achieving safety. MACPO reaches safety via *hard* constraints and backtracking line search, while MAPPO-Lagrangian maintains a rather *soft* safety awareness by performing gradient descent-ascent on the PPO-clip objective.

Figures 5.1-5.6 show cost and reward performance comparisons between MACPO, MAPPO-Lagrangian, MAPPO [276], IPPO [259], and HAPPO [131] algorithms on challenging Safe MAMuJoCo tasks. The experiments reveal that both MACPO and MAPPO-Lagrangian quickly learn to satisfy safety constraints, and keep their explorations within the feasible policy space. This stands in contrast to IPPO, MAPPO, and HAPPO which largely violate the constraints thus being unsafe. Furthermore, our algorithms achieve comparable reward scores; both methods are often better than IPPO. In general, the performance (in terms of reward) of MAPPO-Lagrangian is better than of MACPO; moreover, MAPPO-Lagrangian outperforms the unconstrained MAPPO on challenging *Ant* tasks (Figure 5.2). We note that on none of the tasks the reward of HAPPO was exceeded, though the algorithm is unsafe.

5.5.2 Safe Multi-Agent Robosuite Tasks

In this subsection we describe the Safe Multi-Agent Robosuite environment and provide the results of our experiments conducted on it.

We study the performance of our methods in Safe TwoArmPegInHole in comparison to the aforementioned baselines. Notably, from Figure 5.7 we learn that IPPO completely fails to increase reward in TwoArmPegInHole 2x7, which suggests that the environment’s difficulty is higher than that of Safe MAMuJoCo.

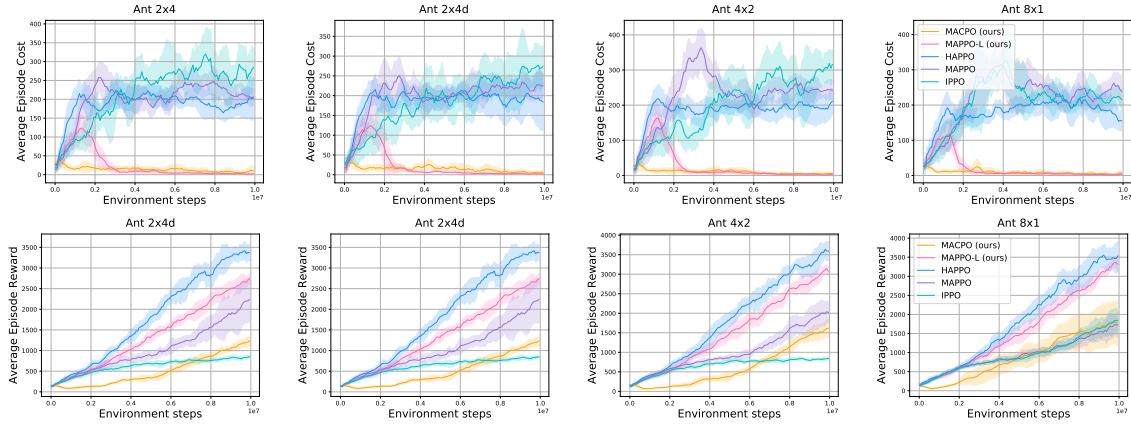


Figure 5.2: Performance comparisons on Safe Ant task 1.0 in terms of cost and reward. The safety constraint values are: 0.2 for 2-agent and 4-agent Ants, and 1 for 8-agent Ants. Each subfigure represents a different robot, within each subfigure, four task setups in terms of multi-agent control are considered, and for each task we plot the cost curves (the lower the better) in the upper row and the reward curves (the higher the better) in the bottom row.

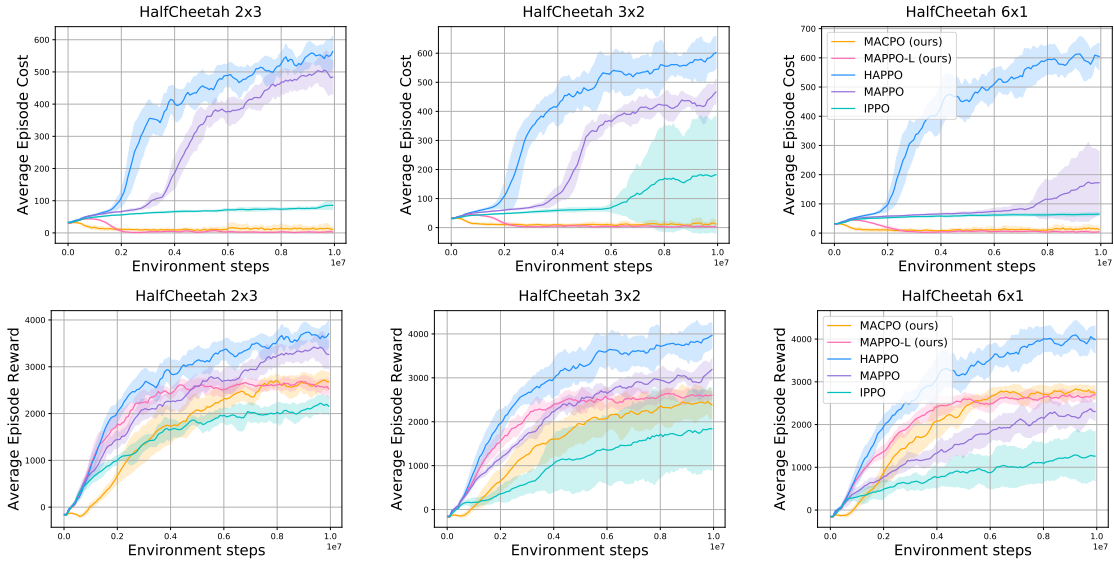


Figure 5.3: Performance comparisons on tasks of Safe HalfCheetah task 1.0 in terms of cost and reward. The safety constraint value is 5. Each subfigure represents a different robot, within each subfigure, three task setups in terms of multi-agent control are considered, and for each task we plot the cost curves (the lower the better) in the upper row and the reward curves (the higher the better) in the bottom row.

5.5.3 Safe Multi-Agent Isaac Gym Tasks

Safe Multi-Agent Isaac Gym (Safe MAIG) is developed on top of Isaac Gym [164]—a high performance, GPU-based platform for robotics tasks that utilises the Nvidia PhysX [186] engine. Again, we employ this environment to compare our methods to other MARL algorithms.

Again, the study reveals that the performance of our methods in terms of reward is competitive with the SOTA MARL methods, meanwhile being safe. Furthermore, as Figures 5.8-5.9 show, MACPO learns to meet the safety constraints faster than MAPPO-Lagrangian. The latter algorithm, however, performs better in terms of reward.

Moreover, in Figure 5.9 left, there is a peak at the end of the MAPPO-L curve. Since

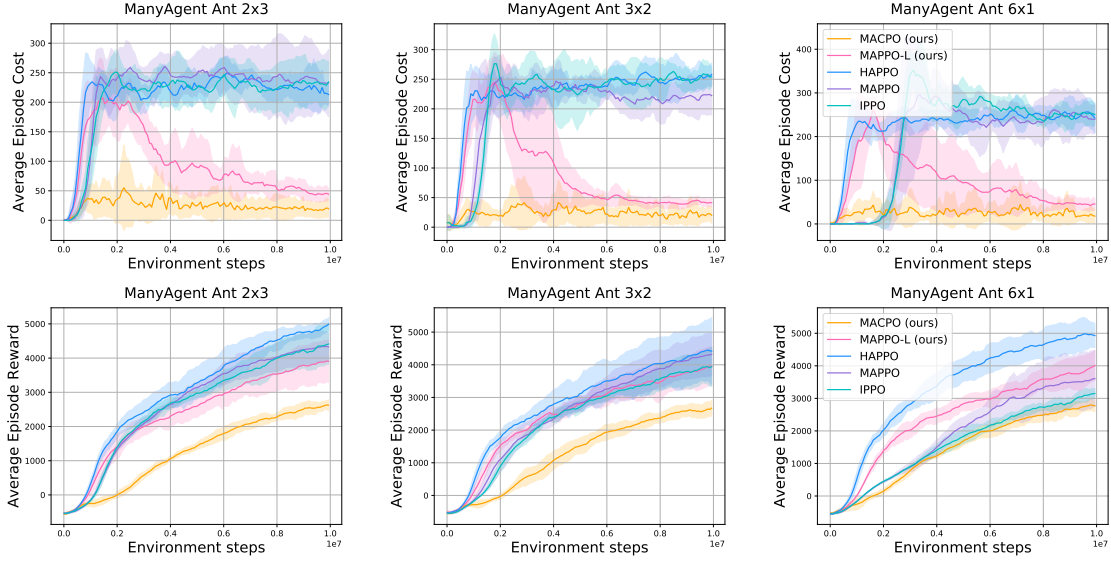


Figure 5.4: Performance comparisons on Safe ManyAgent Ant task 2.1 in terms of cost and reward. The safety constraint value is set to 10. Each subfigure represents a different robot, within each subfigure, three task setups in terms of multi-agent control are considered, and for each task we plot the cost curves (the lower the better) in the upper row and the reward curves (the higher the better) in the bottom row.

MAPPO-L is a soft constraint algorithm, when a safety violation happens, there is a delay to ensure safety, unlike MACPO, which can ensure hard constraints to achieve safety; MAPPO-L is also sensitive to the learning rate. Although there are some disadvantages to the type of MAPPO-Lagrangian methods, currently, it is a popular and efficient algorithm in the safe RL community [94].

5.5.4 Sensitivity Analysis for Safety-Bound-Parameters Tuning

We analyse the sensitivity of MACPO to changes in the size of safety bound (*bound c*) on Figure 5.10. We learn that with that the algorithm’s *effectiveness* does not change under different safety levels. However, with the stricter safety constraints, the reward performance of MACPO decreases.

5.6 Summary of Safe Multi-Robot Learning

In this paper, we have made a step towards solving multi-robot control problems with safety constraints via safe multi-agent reinforcement learning. We introduced the first general formulation of the safe MARL problem and analysed it theoretically. Central to our findings is a safe multi-agent policy iteration procedure that attains theoretically-justified monotonic improvement and constraints satisfaction guarantee at every iteration. As further outcomes to our analysis, we proposed two practical algorithms: MACPO and MAPPO-Lagrangian. To demonstrate their effectiveness, as well as to increase the range of testbeds for Safe MARL methods, we introduced three new benchmark suites of Safe MAMuJoCo, Safe MARobosuite and Safe MAIG. We used them to compare our methods against strong MARL baselines, over which they showed a significant advantage in terms of safety, and comparable performance in terms of the reward. We believe that our findings will contribute to the rise and applicability

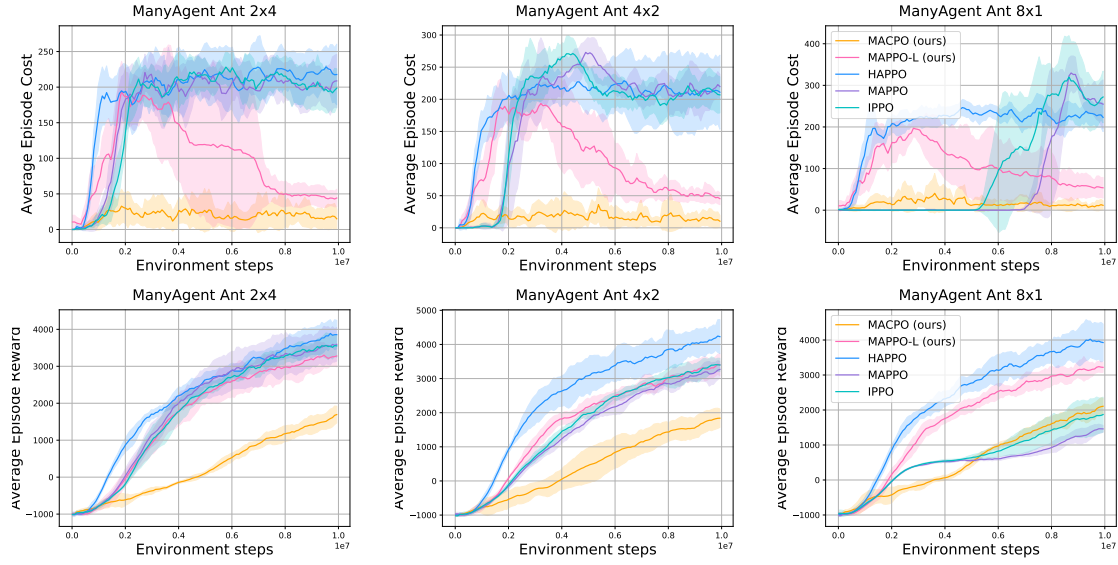


Figure 5.5: Performance comparisons on Safe ManyAgent Ant task 2.2 in terms of cost and reward. The safety constraint value is set to 10. Each subfigure represents a different robot, within each subfigure, three task setups in terms of multi-agent control are considered, and for each task we plot the cost curves (the lower the better) in the upper row and the reward curves (the higher the better) in the bottom row.

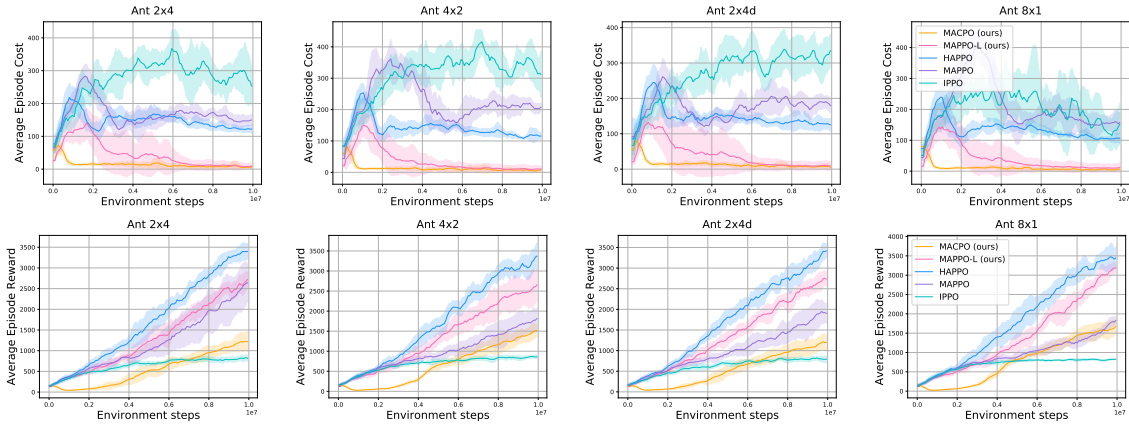


Figure 5.6: Performance comparisons on Safe Ant task 2.1 in terms of cost and reward. The safety constraint value is 1. Each subfigure represents a different robot, within each subfigure, four task setups in terms of multi-agent control are considered, and for each task we plot the cost curves (the lower the better) in the upper row and the reward curves (the higher the better) in the bottom row.

of many safe MARL methods for multi-robot control.

5.7 Details of Settings for Experiments of Safe Multi-Robot Learning

In this section, we provide the details of settings for our experiments. The code is available at: <https://sites.google.com/view/aij-safe-marl/>.

hyperparameters	value	hyperparameters	value
gain	0.01	optimizer	Adam
activation	ReLU	optim eps	1e-5
std x coef	1	max grad norm	10
std y coef	0.5	actor network	mlp

Table 5.1: In the Safe MAMuJoCo, Safe MARobosuite and Safe MAIG domains, common hyperparameters used for MAPPO-Lagrangian, MAPPO, HAPPO, IPPO, and MACPO.

hyperparameters	value	hyperparameters	value
critic lr	5e-3	hidden layer	1
gamma	0.99	eval episodes	32
hidden layer dim	64	episode length	[1000, 2000]
num mini-batch	40	batch size	16000
training threads	4	rollout threads	16

Table 5.2: In the Safe MAMuJoCo and Safe MARobosuite domains, common hyperparameters used for MAPPO-Lagrangian, MAPPO, HAPPO, IPPO, and MACPO (episode length: 1000 used for Safe MAMuJoCo, 2000 used for Safe MARobosuite).

hyperparameters	value	hyperparameters	value
critic lr	[1e-3 ^a , 5e-4 ^b]	hidden layer	2
gamma	0.96	episode length	8
training env number	[512 ^a , 2048 ^b]	eval episodes	10000
rollout threads	80	hidden layer dim	512
num mini-batch	1	num env steps	[100 m ^a , 50 m ^b]

Table 5.3: In the Safe MAIG domains, common hyperparameters used for MAPPO-Lagrangian, MAPPO, HAPPO, IPPO, and MACPO (m denotes million, ^a indicates the parameter is used for ShadowHandOver task, ^b indicates the parameter is used for ShadowHandReOrientation task).

Algorithms	MAPPO-L	MAPPO	HAPPO	IPPO	MACPO
ppo epoch	5	5	5	5	/
ppo-clip	0.2	0.2	0.2	0.2	/
fraction	/	/	/	/	0.5

Table 5.4: In the Safe MAMuJoCo, Safe MARobosuite and Safe MAIG domains, different hyperparameters used for MAPPO-Lagrangian, MAPPO, HAPPO, IPPO, and MACPO.

Algorithms	MAPPO-L	MAPPO	HAPPO	IPPO	MACPO
actor lr	9e-5	9e-5	9e-5	9e-5	/
kl-threshold	/	/	/	/	[0.0065, 1e-3]
lamda lagr	0.78	/	/	/	/
lagrangian coef rate	1e-7	/	/	/	/
fraction coef	/	/	/	/	0.27

Table 5.5: In the Safe MAMuJoCo and Safe MARobosuite domains, different hyperparameters used for MAPPO-Lagrangian, MAPPO, HAPPO, IPPO, and MACPO (kl-threshold: 0.0065 used for Safe MAMuJoCo, 1e-3 used for Safe MARobosuite).

Parameters	actor lr	kl-threshold	fraction coef
MAPPO-L	[3e-5 ^a , 5e-4 ^b]	/	/
MAPPO	[3e-5 ^a , 5e-4 ^b]	/	/
HAPPO	[3e-5 ^a , 5e-4 ^b]	/	/
IPPO	[3e-5 ^a , 5e-4 ^b]	/	/
MACPO	/	[67e-4 ^a , 65e-4 ^b]	[0.29 ^a , 0.27 ^b]

Table 5.6: In the Safe MAIG domains, different hyperparameters used for MAPPO-Lagrangian, MAPPO, HAPPO, IPPO, and MACPO, *a* indicates the parameter is used for ShadowHandOver task, *b* indicates the parameter is used for ShadowHandReOrientation task.

task	value	task	value
Ant(2x4) 1.0	0.2	Ant(4x2) 1.0	0.2
Ant(2x4d) 1.0	0.2	HalfCheetah(2x3) 1.0	5
HalfCheetah(3x2) 1.0	5	HalfCheetah(6x1) 1.0	5
ManyAgent Ant(2x3) 1.0	1	ManyAgent Ant(3x2) 1.0	1
ManyAgent Ant(6x1) 1.0	1	TwoArmPegInHole(2x7) 1.0	30
ManyAgent Ant(2x3) 2.1	10	ManyAgent Ant(3x2) 2.1	10
ManyAgent Ant(6x1) 2.1	10	ManyAgent Ant(2x3) 2.2	10
ManyAgent Ant(3x2) 2.2	10	ManyAgent Ant(6x1) 2.2	10
Ant(2x4) 2.1	1	Ant(4x2) 2.1	1
Ant(2x4d) 2.1	1	Ant(8x1) 2.1	1
ShadowHandOver(2x6)	2.5	ShadowHandReOrientation(4x3)	0.1

Table 5.7: Safety bound in the Safe MAMuJoCo, Safe MARobosuite Safe MAIG domains.

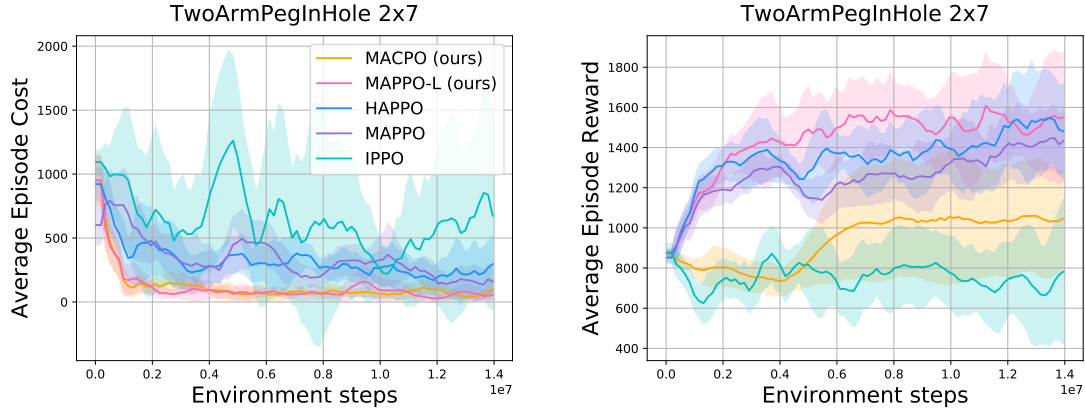


Figure 5.7: Performance comparisons on Safe TwoArmPegInHole 2x7 task, with the safety constraint set to 30.

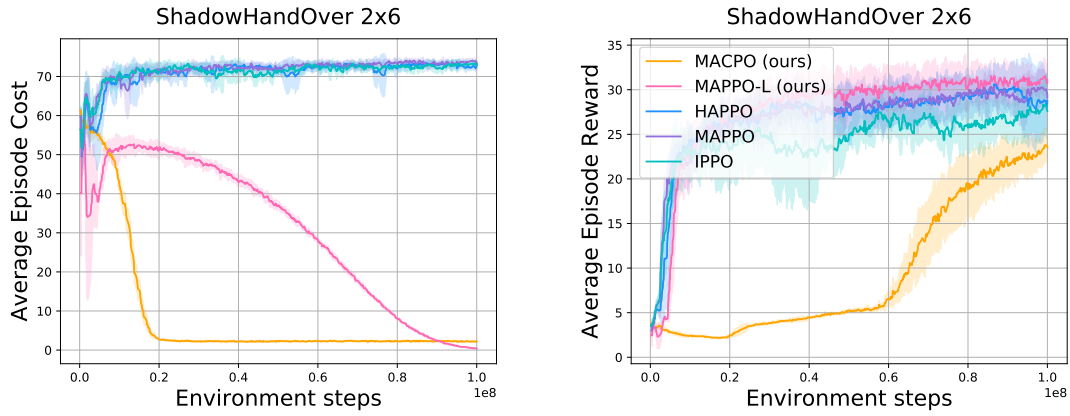


Figure 5.8: ShadowHandOver 2x6 task: performance comparisons Safe ShadowHandOver 2x6 task in terms of cost and reward, the safety constraint value is set to 2.5. Our algorithms are the only ones that learn the safety constraints, while achieving satisfying performance in terms of the reward.

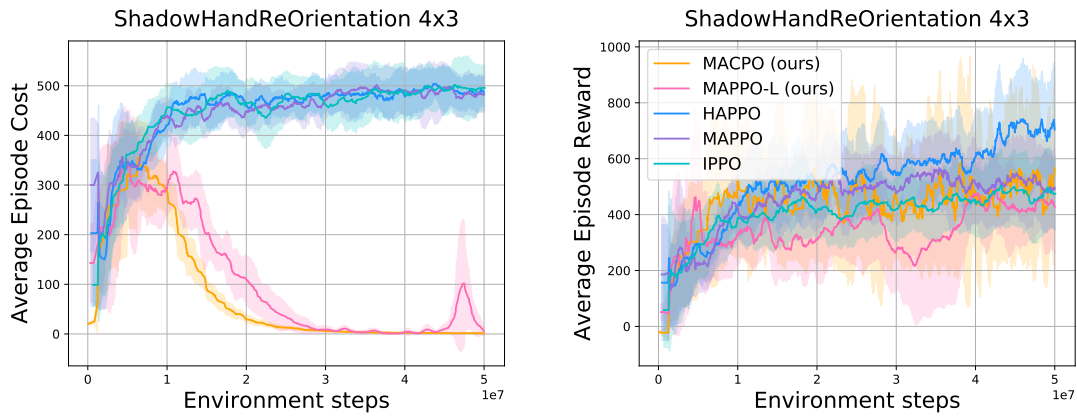


Figure 5.9: ShadowHandReOrientation 4x3 task: performance comparisons on Safe ShadowHandReOrientation 4x3 task in terms of cost and reward. The safety constraint value is set to 0.1.

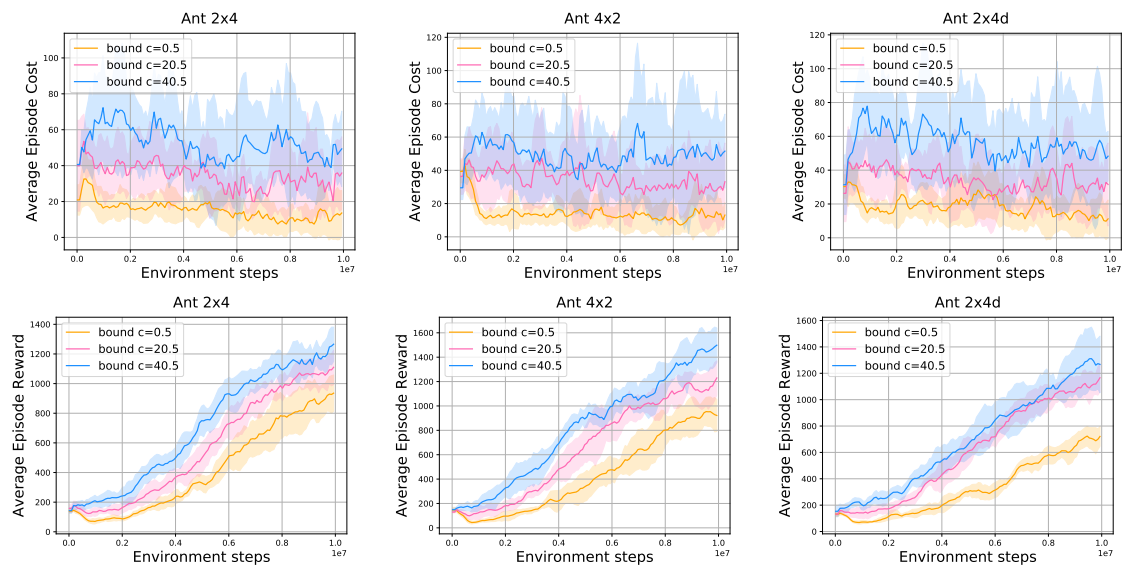


Figure 5.10: Ant 2x4, 4x2, 2x4d tasks 2.2, MACPO with different safety bound in terms of costs and rewards.

5.8 Solution to the Constrained Optimisation Problem in MACPO

Theorem 7. *The solution to the following problem*

$$\begin{aligned} \mathbf{p}^* &= \min_{\mathbf{x}} \mathbf{g}^T \mathbf{x} \\ \text{s.t. } &\mathbf{b}^T \mathbf{x} + c \leq 0 \\ &\mathbf{x}^T \mathbf{H} \mathbf{x} \leq \delta, \end{aligned}$$

where $\mathbf{g}, \mathbf{b}, \mathbf{x} \in \mathbb{R}^n, c, \delta \in \mathbb{R}, \delta > 0, \mathbf{H} \in \mathbb{S}^n$, and $\mathbf{H} \succ 0$. When there is at least one strictly feasible point, the optimal point $\mathbf{x}^*$ satisfies:

$$\mathbf{x}^* = -\frac{1}{\lambda_*} \mathbf{H}^{-1} (\mathbf{g}^T + v_* \mathbf{b})$$

where λ_* and v_* are defined by

$$\begin{aligned} v_* &= \left(\frac{\lambda_* c - r}{s} \right)_+ \\ \lambda_* &= \arg \max_{\lambda \geq 0} \begin{cases} f_a(\lambda) \triangleq \frac{1}{2\lambda} \left(\frac{r^2}{s} - q \right) + \frac{\lambda}{2} \left(\frac{c^2}{s} - \delta \right) - \frac{rc}{s} & \text{if } \lambda c - r > 0 \\ f_b(\lambda) \triangleq -\frac{1}{2} \left(\frac{q}{\lambda} + \lambda \delta \right) & \text{otherwise} \end{cases} \end{aligned}$$

where $q = \mathbf{g}^T \mathbf{H}^{-1} \mathbf{g}$, $r = \mathbf{g}^T \mathbf{H}^{-1} \mathbf{b}$, and $s = \mathbf{b}^T \mathbf{H}^{-1} \mathbf{b}$.

Furthermore, let $\Lambda_a \triangleq \{\lambda \mid \lambda c - r > 0, \lambda \geq 0\}$, and $\Lambda_b \triangleq \{\lambda \mid \lambda c - r \leq 0, \lambda \geq 0\}$. The value of λ_* satisfies

$$\lambda_* \in \left\{ \lambda_*^a \triangleq \text{Proj} \left(\sqrt{\frac{q - r^2/s}{\delta - c^2/s}}, \Lambda_a \right), \lambda_*^b \triangleq \text{Proj} \left(\sqrt{\frac{q}{\delta}}, \Lambda_b \right) \right\} \quad (5.32)$$

where $\lambda_* = \lambda_*^a$ if $f_a(\lambda_*^a) > f_b(\lambda_*^b)$ and $\lambda_* = \lambda_*^b$ otherwise, and $\text{Proj}(a, S)$ is the projection of a point x on to a set S . Note the projection of a point $x \in \mathbb{R}$ onto a convex segment of $\mathbb{R}, [a, b]$, has value $\text{Proj}(x, [a, b]) = \max(a, \min(b, x))$.

Proof. See [5] (Appendix 10.2). □

6

Safe Robust Robot Reinforcement Learning

In this chapter, we develop a robust and safe RL methodology grounded in manifold space. Initially, we construct a constrained manifold space, taking safety constraints into consideration. We then propose a robust safe RL approach, supported by theoretical analysis, based on Value-at-Risk (VaR) and Conditional Value-at-Risk (CVaR), in order to enhance the robustness of safety. Our methodology is designed to ensure safety within stochastic constraint environments. Following the theoretical analysis, we develop a practical, safe algorithm to search for a robust safe policy on stochastic constraint manifolds (ROSCOM). We evaluate the effectiveness of our approach through circular motion and air-hockey tasks. Our experiments demonstrate that ROSCOM outperforms existing baselines in terms of both reward and safety.

6.1 Introduction of Safe Robust Robot Reinforcement Learning

Safe RL methods are leveraged to ensure learning safety by incorporating safety requirements during the learning process and ensuring that the decisions made by the RL agent do not lead to hazardous outcomes [86, 89, 94]. For instance, numerous SOTA methods have been proposed to ensure RL safety, including CPO [5], PCPO [271], RCPO [241], MACPO [89], and MAPPO-Lagrangian [89]. However, these approaches are not applicable to situations where the robot constraints are stochastic. Such scenarios are frequently encountered in real-world applications with high-dimensional space, like robotics and autonomous driving. The robot's observations may exhibit uncertainty due to noisy sensors, while the robot's constraints may be uncertain owing to the presence of other agents, such as humans. Thus, to ensure the safety of a robot and its environment, it is essential to consider stochastic constraints during robot learning in high-dimensional space [91].

In this work, we aim to address the critical question: *How can we guarantee RL safety with stochastic constraints in high-dimensional space?* To ensure safety in stochastic environments with high-dimensional space, we need to calculate safety bounds and construct constraint manifolds that incorporate these bounds. Manifolds are highly effective for managing high-dimensional data, a fact that is well-supported by numerous studies[82, 111, 240]. With the constraint manifolds, we can search for a safe policy for robot learning, where the constraint manifold is well-suited for representing high-dimensional constraints. The contributions of this work are as follows:

- We introduce a problem formulation that considers stochastic constraints on manifolds. Specifically, the stochastic constraints are incorporated into the state space based on

manifolds.

- We offer theoretical safety bounds on the stochastic constraint manifold by utilizing Value-at-Risk (VaR) and Conditional Value-at-Risk (CVaR) [53, 275].
- We propose an algorithm for searching a robust safe policy on a stochastic constraint manifold (ROSCOM) and demonstrate its effectiveness compared to several state-of-the-art safe RL algorithms.

6.2 Related Techniques of Safe Robust Reinforcement Learning

Deploying RL in real-world applications necessitates the assurance of safety for both the agent and the environment, in addition to robustness in the face of uncertainties. Researchers have proposed a variety of safe and robust RL methods to apply RL in safety-critical environments, such as autonomous driving and robotics [86, 88, 94, 155]. Despite these efforts, there remains a need for further development of RL methods that can address safety and robustness challenges concurrently. This section provides a brief analysis of the current state of research in the domain of safe and robust RL.

6.2.1 Safe Reinforcement Learning.

In recent years, research on safe RL has gained significant attention due to its importance in addressing the safety concerns in various applications [94]. Safe RL can be formulated as a constrained optimization problem with methods falling into three main categories: constrained state space, constrained action space, and constrained cumulative cost.

The first category comprises methods that leverage constrained state space as a safe state space [25, 231, 247, 254]. These methods often incorporate Gaussian Process models to estimate safe state space during exploration [25, 231, 247, 254]. Another notable approach is ATACOM [149], which projects exploration states onto a constrained manifold. Our method also belongs to this category.

The second category includes methods that employ constrained action space as a safe action space [51, 52, 80, 128, 142, 168]. For instance, some methods use temporal logic verification to ensure the safety of actions during exploration [142], while others rely on Lyapunov functions to constrain the action space with energy functions [51, 52, 128].

The third category focuses on optimizing safe policies based on cumulative safety cost [5, 89, 241, 271]. Examples include CPO [5] for single-agent settings and MACPO [89] and MAPPO-Lagrangian [89] for multi-agent settings. These methods demonstrate the versatility of Safe RL approaches in addressing safety concerns across various domains and settings.

6.2.2 Robust Reinforcement Learning.

Existing robust RL methods tackle three types of uncertainties: reward uncertainty [141, 143], transition uncertainty [47, 55, 66, 145, 156, 183, 210, 287], and observation uncertainty [85, 161, 256, 284, 285].

To address reward uncertainty, Liang *et al.* [143] proposed a human-preferences-based learning method, while Li *et al.* [141] developed a meta-learning approach that learns uncertain rewards using normalized maximum likelihood.

For transition uncertainty, Chua *et al.* [55] introduced a probabilistic dynamics model to balance the trade-off between model-based and model-free RL methods. Zhang *et al.* [287] employed a natural player to disrupt the multi-agent system's transitions, similar to an adversarial agent, enabling the ego player to enhance its performance in adversarial environments.

Regarding observation uncertainty, Lutjens *et al.* [161] presented a certified adversarial robustness method for handling uncertainties during exploration. Zhang *et al.* [285] proposed a robust policy regularizer for uncertain observations in both discrete and continuous environments. These methods demonstrate the breadth of approaches in robust RL for addressing various uncertainties, thus facilitating the development of more reliable and resilient agents in diverse application scenarios.

The existing methods primarily focus on addressing either the safety or robustness aspect of RL, but not both simultaneously. The challenge of robust and safe RL lies in ensuring safety in uncertain environments, such as those with uncertain observations and constraints. The work of Liu *et al.* [155] is the closest to addressing this challenge, as they proposed a robust and safe RL method in an adversarial setting where two attackers disturb agent behaviors. However, the adversarial setting may limit the practicality of their method in real-world applications.

In contrast to Liu *et al.* [155], our approach ensures RL safety under stochastic constraints by employing a constraint manifold, which is capable of handling high-dimensional data in real-world environments. The stochastic constraints are generated by a stochastic function rather than an adversarial agent. Unlike Liu *et al.* [155], who employ neural networks to model the adversarial agent's behaviors. Lastly, our work provides theoretical safety bounds based on the constraint manifold, ensuring safety in stochastic constraints.

Our method focuses on the observation uncertainty, where the state space is disturbed by some noise, and we need to ensure safety via safety bounds. By addressing both safety and robustness in RL simultaneously, our method paves the way for more reliable and resilient agents that can operate effectively in safety-critical applications and stochastic-constrained environments.

6.3 Problem Formulation of Safe Robust Reinforcement Learning

A robust safe RL problem [94] can be seen as a standard MDP with a constraint set under stochastic constraints, to illustrate this problem, we consider a tuple $(\mathcal{S}, \mathcal{A}, P, r, \rho_0, \gamma, c_i, \epsilon_i)$, where $\mathcal{S}$ is a state space, $\mathcal{A}$ is an action space, $P : \mathcal{S} \times \mathcal{A} \times \mathcal{S} \rightarrow [0, 1]$ is a transition function, r denotes the reward, ρ_0 is the initial state distribution, γ is a discount factor, $c_i : \mathcal{S} \rightarrow \mathbb{R}$ denotes a deterministic state-constraint function of constraint i , $\forall i = 1, \dots, N$, $\epsilon_i : \mathcal{S} \rightarrow \mathbb{R}$ denotes a stochastic state-constraint function of constraint i .

In robust safe RL, the goal of an optimal policy π is to maximize the reward while ensuring safety by selecting an action a under stochastic constraints. $\tau \sim \pi$ is a trajectory, $\tau = (s_0, a_0, s_1, \dots)$, which depends on π , $s_0 \sim \rho_0$, $a_t \sim \pi(\cdot | s_t)$, $s_{t+1} \sim P$, where the state $s_t \in \mathcal{S}$ at time step t , the action $a_t \in \mathcal{A}$ at time step t , $t \in \{0, 1, \dots, T\}$, T is the episode length.

We formulate the robust safe RL problem as an MDP problem as shown in Equation (6.1), in which the stochastic constraints are taken into account. Figure 6.1 shows an example of the stochastic constraints. In this Equation, we need to search for a policy to maximize the discounted cumulative reward value while ensuring learning safety. Specifically, the safety constraints consist of deterministic constraints $c_i(s_t)$ and stochastic constraints $\epsilon_i(s_t)$ at time step t , the stochastic constraints $\epsilon_i(s_t)$ are from some uncertain noises due to imperfect observation. The aggregate of these constraints, $c_i(s_t)$ plus $\epsilon_i(s_t)$, need to remain below a predefined threshold δ with a probability of η . Particularly, for all states s_t , if the constraints

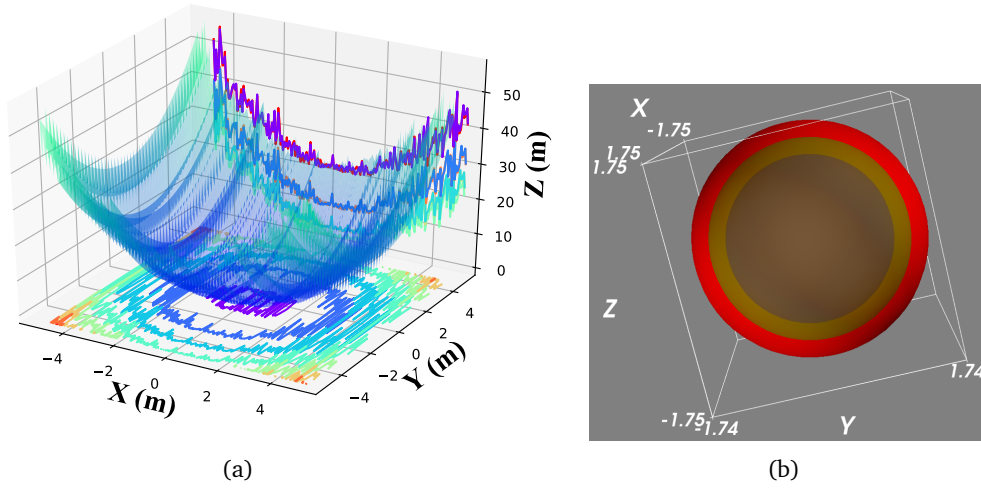


Figure 6.1: Schematic diagram of stochastic constraints and safety bound. (a). 3D stochastic constraints, Gaussian noise in the x, y, and z directions; (b). 3D safety bound, the areas of dark brown and red denote the new safe space, and the area of light brown denotes the uncertain space.

$c_i(s_t)$ and $\epsilon_i(s_t)$ are equal to zero, then states s_t are the safe set (safe set indicates that we can ensure robot learning safety within these states), that is the safe set = $\{s_t | c_i(s_t) = 0, \epsilon_i(s_t) = 0\}$. Conversely, for all states within the unsafe set, $c_i(s_t)$ and $\epsilon_i(s_t)$ are positive real values, expressed as $\mathbb{R}^+$. That is the unsafe set = $\{s_t | c_i(s_t) \in \mathbb{R}^+, \epsilon_i(s_t) \in \mathbb{R}^+\}$. For each type of constraint i , there corresponds a safety bound δ_i and a safety probability η_i . For simplicity, we denote these as δ and η , respectively.

$$\begin{aligned} \max_{\pi} \quad & \mathbb{E}_{\tau \sim \pi} \left[\sum_t^T \gamma^t r(s_t, a_t) \right], \\ \text{s.t.} \quad & \Pr(|c_i(s_t) + \epsilon_i(s_t)| \leq \delta) > \eta. \end{aligned} \quad (6.1)$$

6.4 Method

We first present the definitions of state constraint manifolds. Subsequently, we provide the theoretical safety bounds based on VaR and CVaR to search for safe policies. Sections 6.4.1 is to build a stochastic constrained manifold, and Section 6.4.2 aims to provide safety bounds on a stochastic constrained Manifold. Lastly, Section 6.4.3 presents a practical algorithm for searching robust safe policies within a stochastic constraint manifold.

6.4.1 State Constraints on a Manifold

Similar to ATACOM [149], in the high-dimensional state space $\mathcal{S}$ (the state variable $s \in \mathcal{S}$) is consisted of two sets, a controllable set $q \in \mathcal{Q} \in \mathbb{R}^Q$ and an uncontrollable set $x \in \mathcal{X}$, $s = [q, x]^\top$. In this study, constraints are defined on the controllable set, $\Pr(|c_i(q) + \epsilon_i(q)| < \delta) > \eta$, where $c_i = [f_i, g_i] \in \mathbb{R}^{F+G}$ denotes constraints, equality constraints $f_i = 0$, $f_i \in \mathbb{R}^F$ and inequality constraints $g_i < 0$, $g_i \in \mathbb{R}^G$, $\epsilon_i \in \mathbb{R}^{F+G}$ denotes stochastic constraints, $\delta \in \mathbb{R}^{F+G}$ denotes a set of safety bounds, for simplicity, we use $\eta \in \mathbb{R}^{F+G}$ to denote a set of safety

probability $\eta^{o,o \in \{1,2,\dots,F+G\}}$, f_i to denotes a set of equality constraints $f_i^{o_1,o_1 \in \{1,2,\dots,F\}}$, g_i to denotes a set of equality constraints $g_i^{o_2,o_2 \in \{1,2,\dots,G\}}$. Note, when comparing values of a high-dimensional data set, we assess each individual element within the data set.

Definition 4. *If the constraints are differentiable, we can have the following constraint manifolds. $\mathcal{M}_1$ is the equality constraints manifold with stochastic constraints, $\mathcal{M}_2$ is the inequality constraints manifold with stochastic constraints, as shown in Equation (6.2):*

$$\left[\begin{array}{l} \mathcal{M}_1 = \{|f_i(\mathbf{q}) + \epsilon_{f_i}(\mathbf{q})| - \delta = \mathbf{0}\} \\ \mathcal{M}_2 = \{|g_i(\mathbf{q}) + \epsilon_{g_i}(\mathbf{q})| - \delta + \frac{1}{2}\mu_s^2 = \mathbf{0}\} \end{array} \right] \Rightarrow \left[\begin{array}{l} \mathcal{M}_{1V} = \{f_{iV}(\mathbf{q}) = \mathbf{0}\} \\ \mathcal{M}_{2V} = \{g_{iV}(\mathbf{q}) + \frac{1}{2}\mu^2 = \mathbf{0}\} \end{array} \right] \quad (6.2)$$

In Equation (6.2), $\mathcal{M}_{1V}$ and $\mathcal{M}_{2V}$ represent the manifolds of equality and inequality constraints with VaR (Value at Risk) bounds, respectively. In the following section, we will describe how to compute these manifolds to ensure safety. $\epsilon_{f_i} \in \mathbb{R}^F$ denotes the stochastic constraints on the equality constraint manifold, $\epsilon_{g_i} \in \mathbb{R}^G$ denotes the stochastic constraints on the inequality constraint manifold, $f_{iV}(\mathbf{q})$ and $g_{iV}(\mathbf{q})$ denotes equality constraints and inequality constraints with VaR safety bounds considering stochastic constraints. To construct the safety manifold under inequality constraints, the slack variables $\mu_s \in \mathbb{R}^G$ and $\mu \in \mathbb{R}^G$ are leveraged, which are introduced into the constraints [149].

Therefore, we have the new constraint set $\mathbf{c}(\mathbf{q}, \mu_s) \in \mathbb{R}^{F+G}$, as shown in Equation (6.3), $J_{f_{iV}}(\mathbf{q}) \in \mathbb{R}^{F \times Q}$ is the Jacobian matrix of $f_{iV}(\mathbf{q}) \in \mathbb{R}^F$. We have two equality constraint sides when inequality constraints are converted to equality constraints with safety bounds. Thus, we consider two safety sides of the equality constraints, $J_{f_{iV}}^+(\mathbf{q}) \in \mathbb{R}^{F \times Q}$ and $J_{f_{iV}}^-(\mathbf{q}) \in \mathbb{R}^{F \times Q}$. $J_{f_{iV}}^+(\mathbf{q}) \in \mathbb{R}^{F \times Q}$ and $J_{f_{iV}}^-(\mathbf{q}) \in \mathbb{R}^{F \times Q}$ are the Jacobian matrices of the safety bounds for the function $g_{iV}(\mathbf{q}) \in \mathbb{R}^G$.

$$\mathbf{c}(\mathbf{q}, \mu, \dot{\mathbf{q}}, \dot{\mu}) = \begin{bmatrix} J_{f_{iV}}(\mathbf{q})^+ & \mathbf{0} \\ J_{f_{iV}}(\mathbf{q})^- & \mathbf{0} \\ J_{g_{iV}}(\mathbf{q}) & \text{diag}(\mu_s) \end{bmatrix} \begin{bmatrix} \dot{\mathbf{q}} \\ \dot{\mu}_s \end{bmatrix} \quad (6.3)$$

$$= J_c(\mathbf{q}, \mu_s) \begin{bmatrix} \dot{\mathbf{q}} \\ \dot{\mu}_s \end{bmatrix}, \quad (6.4)$$

where $J_c(\mathbf{q}, \mu_s) \in \mathbb{R}^{(2F+G) \times (Q+G)}$ is the Jacobian Matrix, by leveraging SVD [228] and QR [123], we can have $J_c(\mathbf{q}, \mu_s)N_c(\mathbf{q}, \mu_s) = \mathbf{0}$, $N_c(\mathbf{q}, \mu_s) \in \mathbb{R}^{(Q+G) \times (Q-2F)}$ is the null space, which can be seen as the tangent space bases of a constraint manifold.

Remark 1. *Inspired by ATACOM [149], the nullspace of the Jacobian matrix $J_c(\mathbf{q}, \mu_s) \in \mathbb{R}^{(2F+G) \times (Q+G)}$ for the tangent-space bases of the constraint manifold is $N_c(\mathbf{q}, \mu_s) \in \mathbb{R}^{(Q+G) \times (Q-2F)}$, we can search a safe policy at each time step t on the constraint manifold $\mathbf{c}(\mathbf{q}, \mu_s)$.*

In the subsequent section, we will delve into the methodologies and strategies to guarantee safety on a stochastic manifold. The complexities inherent to these manifolds necessitate rigorous approaches to maintain safety, and our discussion will shed light on these critical procedures.

6.4.2 Ensuring Safety with VaR and CVaR on a Stochastic Constraint Manifold

This section presents a theoretical analysis of the safety bounds based on stochastic constraints. First, we give the definition of the safety bounds and the risk measurements. Then, we derive safety bounds for stochastic constraints by leveraging VaR and CVaR [53], [275].

Definition 5. *VaR and CVaR.* For safety constraints $|c_i(\mathbf{q}) + \epsilon_i(\mathbf{q})|$, the value-at-risk (VaR) of $|c_i(\mathbf{q}) + \epsilon_i(\mathbf{q})|$ with confidence level $\eta \in (0, 1)$ is defined as:

$$\text{VaR}_\eta(|c_i(\mathbf{q}) + \epsilon_i(\mathbf{q})|) = \min\{\delta \mid F(\delta) \geq \eta\},$$

where $F(\delta) = P(|c_i(\mathbf{q}) + \epsilon_i(\mathbf{q})| \leq \delta)$ is the cumulative distribution function (CDF), $\epsilon_i(\mathbf{q})$ denotes the stochastic costs. The conditional value-at-risk (CVaR) of $|c_i(\mathbf{q}) + \epsilon_i(\mathbf{q})|$ with confidence level η is defined as the expectation of the η -tail distribution of $|c_i(\mathbf{q}) + \epsilon_i(\mathbf{q})|$ as

$$\text{CVaR}_\eta(\epsilon_i(\mathbf{q})) = \mathbb{E}\{\delta \mid \delta \geq \text{VaR}_\eta(|c_i(\mathbf{q}) + \epsilon_i(\mathbf{q})|)\}.$$

Here, we assume stochastic constraints $\epsilon_i(\mathbf{q})$ subject to a Gaussian distribution N , $Z = \epsilon_i(\mathbf{q}) \sim N(\mu, \sigma^2)$.

6.4.2.1 Safety Bound for Inequality Constraints with Stochastic Noises on a Manifold

We rewrite VaR for inequality constraints as following equations, where $\mathbf{g}_i(\mathbf{q})^a$ denotes the old deterministic inequality constraints, $\mathbf{g}_i(\mathbf{q})$ denotes the new deterministic inequality constraints, δ_g denotes the safety bound for inequality constraints, μ denotes the mean of stochastic constraints $\epsilon_i(\mathbf{q})$, σ denotes the standard deviation of stochastic constraints $\epsilon_i(\mathbf{q})$, η is the safety probability for robot learning.

$$F(\delta_g) = P(\mathbf{g}_i(\mathbf{q})^a + \epsilon_i(\mathbf{q}) \leq \delta_g), \quad (6.5)$$

$$\begin{aligned} \text{VaR}_\eta(\mathbf{g}_i(\mathbf{q})^a + \epsilon_i(\mathbf{q})) &= \min\{\delta_g \mid F(\delta_g) \geq \eta\} \\ &= \min\{\delta_g \mid P(\mathbf{g}_i(\mathbf{q})^a + \epsilon_i(\mathbf{q}) \leq \delta_g) \geq \eta\} \\ &= \min\{\delta_g \mid P\left(\frac{\epsilon_i(\mathbf{q}) - \mu}{\sigma} \leq \frac{\delta_g - \mathbf{g}_i(\mathbf{q})^a - \mu}{\sigma}\right) \geq \eta\} \\ &= \min\{\delta_g \mid \Phi\left(\frac{\delta_g - \mathbf{g}_i(\mathbf{q})^a - \mu}{\sigma}\right) \geq \eta\}. \end{aligned} \quad (6.6)$$

(1) Computing safety bounds for inequality stochastic constraints with VaR on a manifold:

Lemma 11. *The safety bound BV_{g1} on a stochastic inequality constraint manifold via VaR is given by,*

$$\mathbf{g}_i(\mathbf{q})^a + \mathbf{g}_i(\mathbf{q}) \leq \underbrace{-\Phi^{-1}(\eta)\sigma - \mu + \mathbf{g}_i(\mathbf{q})}_{\text{safety bound } BV_{g1}}.$$

Proof. With Equation (6.6), we can have the following safety bound,

$$\begin{aligned} \Phi\left(\frac{\delta_g - \mathbf{g}_i(\mathbf{q})^a - \mu}{\sigma}\right) &\geq \eta \\ \implies \frac{\delta_g - \mathbf{g}_i(\mathbf{q})^a - \mu}{\sigma} &\geq \Phi^{-1}(\eta) \\ \implies \delta_g &\geq \Phi^{-1}(\eta)\sigma + \mathbf{g}_i(\mathbf{q})^a + \mu. \end{aligned} \quad (6.7)$$

We can observe the following equation if we want to ensure total safety with probability η ,

$$\begin{aligned} 0 = \delta_g &\geq \Phi^{-1}(\eta)\sigma + g_i(q)^a + \mu \\ \implies g_i(q)^a &\leq -\Phi^{-1}(\eta)\sigma - \mu. \end{aligned} \quad (6.8)$$

Therefore, the safety bound BV_{g1} via VaR that considered uncertain areas is given as follows:

$$g_i(q)^a + g_i(q) \leq \underbrace{-\Phi^{-1}(\eta)\sigma - \mu + g_i(q)}_{\text{safety bound } BV_{g1}}. \quad (6.9)$$

□

Based on Lemma 11, we can have VaR safety bound for inequality constraints in terms of (a) safety probability, (b) constraints' mean and (c) constraints' standard deviation, as shown in Figure 6.2. It demonstrates the robustness of our safety bound under varying conditions of safety probability (a), mean of constraints (b), and standard deviation of constraints (c). Once we have the safety bounds, we can search for the policy on the tangent space of safety bounds, which helps ensure performance stability. We also provide CVaR safety bound for inequality constraints, for details, see the following Lemma 12.

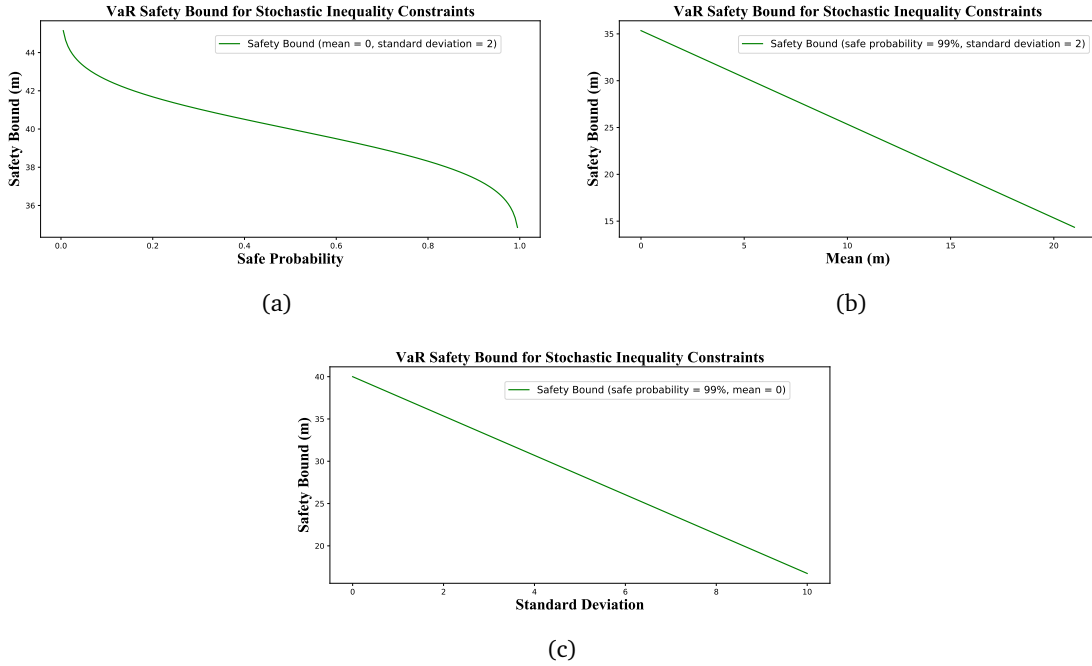


Figure 6.2: VaR safety bounds on a manifold for inequality constraints in terms of (a) safe probability, (b) constraints' mean, and (c) constraints' standard deviation.

(2) Computing safety bounds for inequality stochastic constraints with CVaR on a manifold:

Inspired by the reference [185], since the stochastic constraints subjects to a Gaussian distribution, we can have

$$\phi = \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{1}{2}\left(\frac{x-g_i(q)^a-\mu}{\sigma}\right)^2}, \quad (6.10)$$

$$\begin{aligned}
\text{CVaR}_\eta &= \mathbb{E}[X \mid X \geq \delta_g] = g_i(q)^a + \mu + \sigma \frac{\phi(\Phi^{-1}(\eta))}{1-\eta} \leq 0 \\
\implies g_i(q)^a &\leq -\mu - \sigma \frac{\phi(\Phi^{-1}(\eta))}{1-\eta},
\end{aligned} \tag{6.11}$$

Lemma 12. *The safety bound BC_{g1} on an inequality stochastic constraint manifold via CVaR that considered uncertain areas is given as follows:*

$$g_i(q)^a + g_i(q) \leq \underbrace{-\mu - \sigma \frac{\phi(\Phi^{-1}(\eta))}{1-\eta}}_{\text{safety bound } BC_{g1}} + g_i(q), \tag{6.12}$$

Proof. We have a random vector $\mathbf{X} \in \mathbb{R}^{F+G}$, and $\text{CVaR}_{\delta_g}(\mathbf{X}) = \mathbb{E}(\mathbf{X} \mid \mathbf{X} > \delta_g) = \int_{\delta_g}^{\infty} \mathbf{x} f(\mathbf{x}) d\mathbf{x} / (1 - F(\delta_g))$, where $F(\delta_g)$ is a CDF, $f(\mathbf{x})$ is a Probability Density Function (PDF), and $\delta_g = \text{VaR}_{\delta_g}$, and $P(\mathbf{X} > \delta_g) = 1 - \eta$, $\mathbf{x} \in \mathbb{R}^{F+G}$.

As shown in the first part, $\Phi[(\delta_g - \mu)/\sigma] = \eta$, we have

$$\begin{aligned}
\int_{\delta_g}^{\infty} \mathbf{x} f(\mathbf{x}) d\mathbf{x} &= \int_{-\infty}^{\infty} \mathbf{x} f(\mathbf{x}) d\mathbf{x} - \int_{-\infty}^{\delta_g} \mathbf{x} f(\mathbf{x}) d\mathbf{x} \\
&= \mu - \int_{-\infty}^{\delta_g} \mathbf{x} f(\mathbf{x}) d\mathbf{x} \\
&= \mu - \mu \Phi[(\delta_g - \mu)/\sigma] - \sigma \int_{-\infty}^{(\delta_g - \mu)/\sigma} \mathbf{z} \phi(\mathbf{z}) d\mathbf{z} \\
&= \mu - \mu \eta + \sigma \int_{-\infty}^{(\delta_g - \mu)/\sigma} \phi'(\mathbf{z}) d\mathbf{z} \\
&= \mu(1 - \eta) + \sigma \phi[(\delta_g - \mu)/\sigma] \\
&= \mu(1 - \eta) + \sigma \phi[\Phi^{-1}(\eta)].
\end{aligned}$$

Thus, we have the safety bound,

$$\begin{aligned}
\text{CVaR}_{\delta_g}(\mathbf{X}) &= \frac{\mu(1 - \eta) + \sigma \phi[\Phi^{-1}(\eta)]}{1 - F(\delta_g)} \\
&= \frac{\mu(1 - \eta) + \sigma \phi[\Phi^{-1}(\eta)]}{1 - \eta} \\
&= \mu + \frac{\sigma \phi[\Phi^{-1}(\eta)]}{1 - \eta}.
\end{aligned} \tag{6.13}$$

□

In accordance with Lemma 12, it becomes apparent that we are able to establish a safety bound predicated on CVaR for inequality constraints, and this relationship is explicated with respect to three fundamental components: (a) the safe probability, (b) the mean of the constraints, and (c) the standard deviation of the constraints. The representation of these relationships is provided in Figure 6.3.

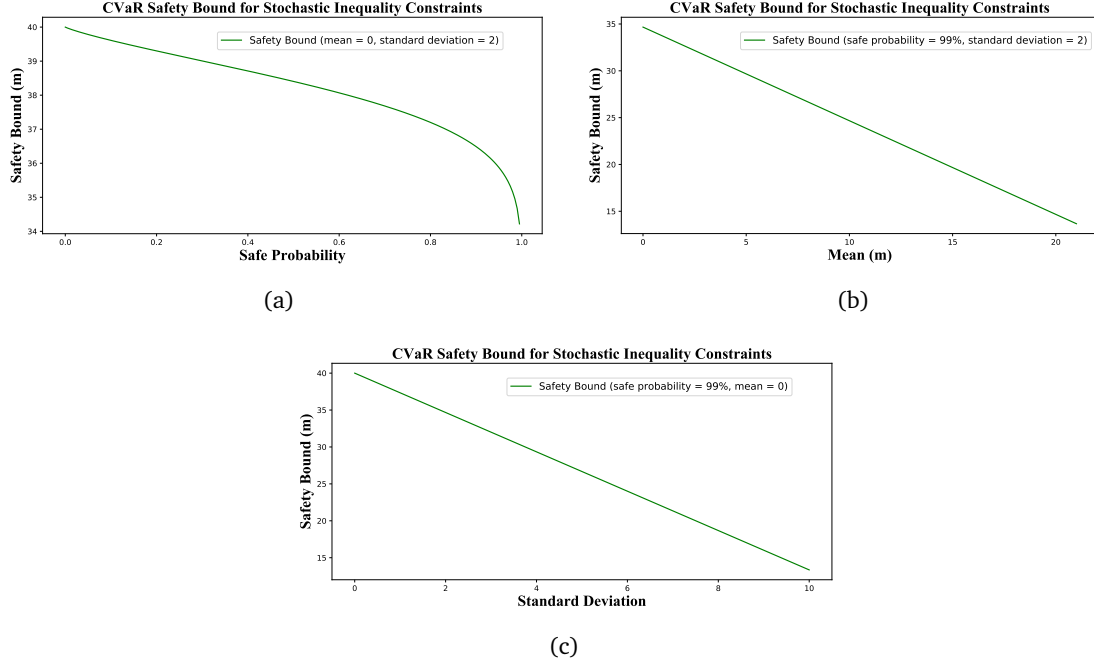


Figure 6.3: CVaR safety bounds on a manifold for inequality constraints in terms of safe probability (a), constraints' mean (b), and constraints' standard deviation (c).

6.4.2.2 Safety Bounds for Equality Constraints with Stochastic Noises on a Manifold

Stochastic equality constraints present a distinct challenge compared to stochastic inequality constraints due to their inherent two-sided constraint structure on a manifold, resulting in the existence of two separate safety bounds. The computation of probabilities for both sides of these equality constraints is a non-trivial task. To address this, we propose a transformation that allows us to reframe the problem of computing two equality safety bounds into two distinct problems centered around the computation of inequality safety bounds. Specifically, we aim for VaR and CVaR safety bounds based on the two inequality safety bounds obtained through this transformation.

Here, we rewrite VaR for equality constraints, $f_i(q)$ denotes equality constraints, as shown in Equations (6.14) and (6.15).

$$\begin{aligned}
 F(\delta_f) &= P(|f_i(q) + \epsilon_i(q)| \leq \delta_f) \\
 &= P(-\delta_f \leq f_i(q) + \epsilon_i(q) \leq \delta_f) \\
 &= P(f_i(q) + \epsilon_i(q) \leq \delta_f) - P(f_i(q) + \epsilon_i(q) \leq -\delta_f) \\
 &= \Phi\left(\frac{\delta_f - f_i(q) - \mu}{\sigma}\right) - \Phi\left(\frac{-\delta_f - f_i(q) - \mu}{\sigma}\right).
 \end{aligned} \tag{6.14}$$

$$\begin{aligned}
 \text{VaR}_\eta(|f_i(q) + \epsilon_i(q)|) &= \min \{ \delta_f \mid F(\delta_f) \geq \eta \} \\
 &= \min \left\{ \delta_f \mid \Phi\left(\frac{\delta_f - f_i(q) - \mu}{\sigma}\right) - \Phi\left(\frac{-\delta_f - f_i(q) - \mu}{\sigma}\right) \geq \eta \right\}.
 \end{aligned} \tag{6.15}$$

For one side safety bound via VaR, $F(\delta_f^a) = P(f_i(q)^a + \epsilon_i(q) > \delta_f^a) < \frac{1-\eta}{2}$. For the other side safety bound via VaR, $F(\delta_f^b) = P(f_i(q)^b + \epsilon_i(q) \leq -\delta_f^b) < \frac{1-\eta}{2}$, δ_f^b is the other side's constrained limit set.

(1) Computing Safety Bounds for Equality Constraints with VaR on a Manifold:

Theorem 8. *The first-side safety bound BV_{f_1} on the equality stochastic constraint manifold via VaR that considers uncertain areas is given by,*

$$f_i(q)^a + f_i(q) < \underbrace{-\Phi^{-1}\left(\frac{1+\eta}{2}\right)\sigma - \mu + f_i(q)}_{\text{safety bound } BV_{f_1}}.$$

The second-side safety bound BV_{f_2} on the equality stochastic constraint manifold via VaR that considers uncertain areas is given by,

$$\underbrace{-\mu - \Phi^{-1}\left(\frac{1-\eta}{2}\right)\sigma + f_i(q)}_{\text{safety bound } BV_{f_2}} < f_i(q)^b + f_i(q).$$

Proof. Based on Equations (6.14) and (6.15), we can prove the first side of safety bounds.

(A). The first side safety bound via VaR:

With the VaR definition on constraint manifolds (as shown in Definition 5) and Equation (6.5), we can have

$$\begin{aligned} F(\delta_f^a) &= P(f_i(q)^a + \epsilon_i(q) > \delta_f^a) < \frac{1-\eta}{2} \\ \implies 1 - P(f_i(q)^a + \epsilon_i(q) \leq \delta_f^a) &< \frac{1-\eta}{2} \\ \implies \frac{1+\eta}{2} &< P\left(\frac{\epsilon_i(q) - \mu}{\sigma} \leq \frac{\delta_f^a - f_i(q)^a - \mu}{\sigma}\right) \\ \implies \frac{1+\eta}{2} &< \Phi\left(\frac{\delta_f^a - f_i(q)^a - \mu}{\sigma}\right) \\ \implies \Phi^{-1}\left(\frac{1+\eta}{2}\right) &< \frac{\delta_f^a - f_i(q)^a - \mu}{\sigma} \\ \implies \Phi^{-1}\left(\frac{1+\eta}{2}\right)\sigma + f_i(q)^a + \mu &< \delta_f^a. \end{aligned} \quad (6.16)$$

Since it's a equality constraint equation, we can have $\delta_f^a = 0$, thus, the condition of uncertain areas of $f_i(q)^a$ is as follows,

$$\Phi^{-1}\left(\frac{1+\eta}{2}\right)\sigma + f_i(q)^a + \mu < \delta_f^a = 0 \quad (6.17)$$

$$\implies f_i(q)^a < -\Phi^{-1}\left(\frac{1+\eta}{2}\right)\sigma - \mu. \quad (6.18)$$

Therefore, the safety bound BV_{f_1} via VaR that considered uncertain areas is given as follows:

$$f_i(q)^a + f_i(q) < \underbrace{-\Phi^{-1}\left(\frac{1+\eta}{2}\right)\sigma - \mu + f_i(q)}_{\text{safety bound } BV_{f_1}}. \quad (6.19)$$

(B). The second side safety bound via VaR:

Similar to the proof of the first side safety bound, we can have

$$\begin{aligned}
 F(\delta_f^b) &= P\left(f_i(q)^b + \epsilon_i(q) \leq -\delta_f^b\right) < \frac{1-\eta}{2} \\
 &\Rightarrow P\left(\frac{\epsilon_i(q) - \mu}{\sigma} \leq \frac{-\delta_f^b - f_i(q)^b - \mu}{\sigma}\right) < \frac{1-\eta}{2} \\
 &\Rightarrow \Phi\left(\frac{-\delta_f^b - f_i(q)^b - \mu}{\sigma}\right) < \frac{1-\eta}{2} \\
 &\Rightarrow -\delta_f^b - f_i(q)^b - \mu < \Phi^{-1}\left(\frac{1-\eta}{2}\right)\sigma \\
 &\Rightarrow -f_i(q)^b - \mu - \Phi^{-1}\left(\frac{1-\eta}{2}\right)\sigma < \delta_f^b.
 \end{aligned} \tag{6.20}$$

The condition of uncertain areas of $f_i(q)^b$ is as follows,

$$\begin{aligned}
 -f_i(q)^b - \mu - \Phi^{-1}\left(\frac{1-\eta}{2}\right)\sigma &< \delta_f^b = 0 \\
 &\Rightarrow -\mu - \Phi^{-1}\left(\frac{1-\eta}{2}\right)\sigma < f_i(q)^b.
 \end{aligned} \tag{6.21}$$

Thus, the safety bound BV_{f_2} via VaR that considered uncertain areas is given as follows:

$$\underbrace{-\mu - \Phi^{-1}\left(\frac{1-\eta}{2}\right)\sigma + f_i(q)}_{\text{safety bound } BV_{f_2}} < f_i(q)^b + f_i(q). \tag{6.22}$$

□

Leveraging the aforementioned Theorem, we are able to establish VaR safety bounds specific to equality constraints. These bounds are characterized by three crucial parameters: the safe probability, the constraints' mean, and the constraints' standard deviation. A detailed representation of this relationship is depicted in Figure 6.4. Furthermore, we present the CVaR safety bounds pertinent to equality constraints. A detailed elaboration of these bounds is introduced in Theorem 9, providing a comprehensive analysis of safety bounds.

(2) Computing Safety Bounds for Equality Constraints with CVaR on a Manifold:

Theorem 9. *For the first side, safety bound via CVaR:*

$$\begin{aligned}
 \text{CVaR}_\eta &= E[X \mid X \geq \delta_f] = f_i(q)^a + \mu + \sigma \frac{\phi\left[\Phi^{-1}\left(\frac{1+\eta}{2}\right)\right]}{\frac{1-\eta}{2}} \\
 &\Rightarrow f_i(q)^a + \mu + \sigma \frac{\phi\left[\Phi^{-1}\left(\frac{1+\eta}{2}\right)\right]}{\frac{1-\eta}{2}} \leq X = 0 \\
 &\Rightarrow f_i(q)^a \leq -\mu - \sigma \frac{\phi\left[\Phi^{-1}\left(\frac{1+\eta}{2}\right)\right]}{\frac{1-\eta}{2}}.
 \end{aligned} \tag{6.23}$$

The first side-safety bound BC_{f_1} on equality stochastic constraint Manifolds via CVaR that considered uncertain areas is given as follows:

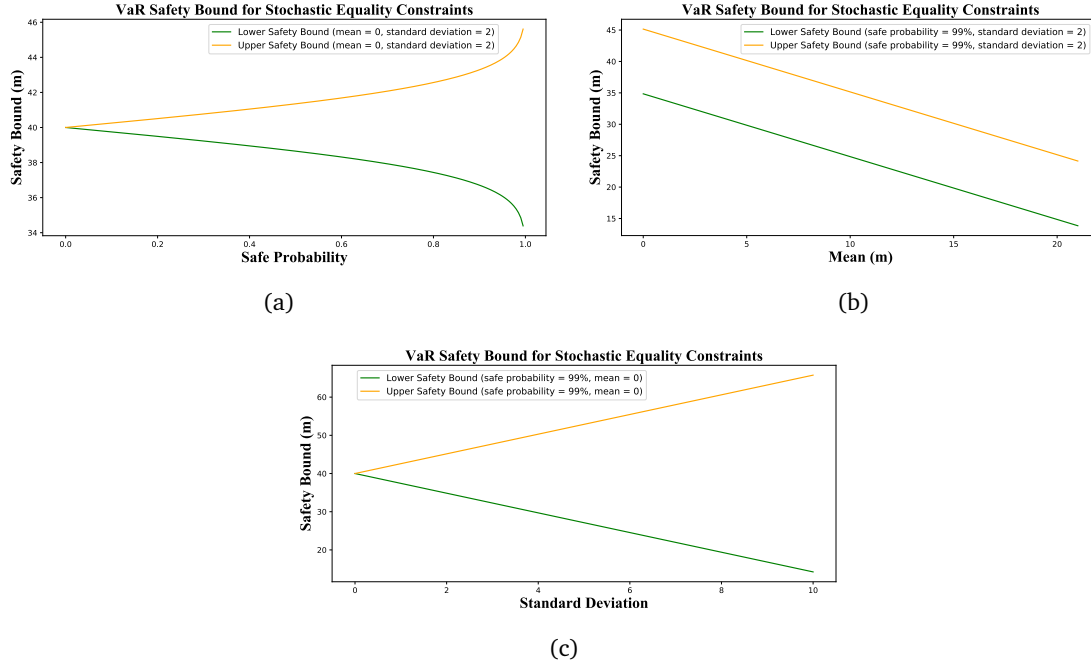


Figure 6.4: VaR safety bounds on a manifold for Equality Constraints in terms of safe probability (a), constraints' mean (b), and constraints' standard deviation (c).

$$f_i(q)^a + f_i(q) \leq \underbrace{-\mu - \sigma \frac{\phi \left[\Phi^{-1} \left(\frac{1-\eta}{2} \right) \right]}{\frac{1-\eta}{2}}}_{\text{safety bound } BC_{f1}} + f_i(q). \quad (6.24)$$

For the second side safety bound via CVaR:

$$\begin{aligned} \text{CVaR}_\eta &= E[X \mid X \leq -\delta_f] \\ &= f_i(q)^b + \mu - \sigma \frac{\phi \left[\Phi^{-1} \left(\frac{1-\eta}{2} \right) \right]}{\frac{1-\eta}{2}} \\ \Rightarrow f_i(q)^b + \mu - \sigma \frac{\phi \left[\Phi^{-1} \left(\frac{1-\eta}{2} \right) \right]}{\frac{1-\eta}{2}} &\leq X = 0 \\ \Rightarrow f_i(q)^b &\leq -\mu + \sigma \frac{\phi \left[\Phi^{-1} \left(\frac{1-\eta}{2} \right) \right]}{\frac{1-\eta}{2}}. \end{aligned} \quad (6.25)$$

The second side-safety bound BC_{f2} on equality stochastic constraint manifolds via CVaR that considered uncertain areas is given as follows:

$$f_i(q)^b + f_i(q) \leq \underbrace{-\mu + \sigma \frac{\phi \left[\Phi^{-1} \left(\frac{1-\eta}{2} \right) \right]}{\frac{1-\eta}{2}}}_{\text{safety bound } BC_{f2}} + f_i(q). \quad (6.26)$$

Proof. On the basis of Lemma 12 and Theorem 8, for a random vector $\mathbf{X}$ within equality constraints on a manifold, CVaR is denoted as $\text{CVaR}_{\delta_f}(\mathbf{X}) = \mathbb{E}(\mathbf{X} | \mathbf{X} > \delta_f) = \int_{\delta_f}^{\infty} \mathbf{x} f(\mathbf{x}) d\mathbf{x} / (1 - F(\delta_f))$, and $\delta_f = \text{VaR}_{\delta_f}$, and $P(\mathbf{X} > \delta_f) = (1 - \eta)/2$ [185].

Since $\mathbf{X} \sim N(\boldsymbol{\mu}, \boldsymbol{\sigma}^2)$, $\mathbf{Z} = (\mathbf{X} - \boldsymbol{\mu})/\boldsymbol{\sigma} \sim N(\mathbf{0}, \mathbf{1})$, the standard normal distribution function Φ specifies $\Phi(\mathbf{z}) = P(\mathbf{Z} \leq \mathbf{z})$ and $1 - \Phi(\mathbf{z}) = P(\mathbf{Z} > \mathbf{z})$.

(A). The first side safety bound via CVaR:

$$\begin{aligned} \int_{\delta_f^a}^{\infty} \mathbf{x} f(\mathbf{x}) d\mathbf{x} &= \int_{-\infty}^{\infty} \mathbf{x} f(\mathbf{x}) d\mathbf{x} - \int_{-\infty}^{\delta_f^a} \mathbf{x} f(\mathbf{x}) d\mathbf{x} \\ &= \boldsymbol{\mu} - \int_{-\infty}^{\delta_f^a} \mathbf{x} f(\mathbf{x}) d\mathbf{x}. \end{aligned}$$

Changing variables with $\mathbf{x} = \boldsymbol{\mu} + \boldsymbol{\sigma} \mathbf{z}$ and $d\mathbf{x} = \boldsymbol{\sigma} d\mathbf{z}$. Based on the change of variables, noting that $\phi(\mathbf{z}) = \boldsymbol{\sigma} f(\boldsymbol{\mu} + \boldsymbol{\sigma} \mathbf{z})$, is a PDF which subjects to a Gaussian distribution, we have

$$\begin{aligned} \int_{-\infty}^{\delta_f^a} \mathbf{x} f(\mathbf{x}) d\mathbf{x} &= \int_{-\infty}^{(\delta_f^a - \boldsymbol{\mu})/\boldsymbol{\sigma}} (\boldsymbol{\mu} + \boldsymbol{\sigma} \mathbf{z}) \phi(\mathbf{z}) d\mathbf{z} \\ &= \boldsymbol{\mu} \int_{-\infty}^{(\delta_f^a - \boldsymbol{\mu})/\boldsymbol{\sigma}} \phi(\mathbf{z}) d\mathbf{z} + \boldsymbol{\sigma} \int_{-\infty}^{(\delta_f^a - \boldsymbol{\mu})/\boldsymbol{\sigma}} \mathbf{z} \phi(\mathbf{z}) d\mathbf{z} \\ &= \boldsymbol{\mu} \Phi\left[(\delta_f^a - \boldsymbol{\mu})/\boldsymbol{\sigma}\right] + \boldsymbol{\sigma} \int_{-\infty}^{(\delta_f^a - \boldsymbol{\mu})/\boldsymbol{\sigma}} \mathbf{z} \phi(\mathbf{z}) d\mathbf{z}. \end{aligned}$$

As shown in the above Equations, $\Phi\left[(\delta_f^a - \boldsymbol{\mu})/\boldsymbol{\sigma}\right] = \eta + (1 - \eta)/2 = (1 + \eta)/2$, and the key here is the observation that the standard normal density function satisfies $\mathbf{z} \phi(\mathbf{z}) = -\phi'(\mathbf{z})$.

$$\begin{aligned} \int_{\delta_f^a}^{\infty} \mathbf{x} f(\mathbf{x}) d\mathbf{x} &= \boldsymbol{\mu} - \boldsymbol{\mu} \Phi\left[(\delta_f^a - \boldsymbol{\mu})/\boldsymbol{\sigma}\right] - \boldsymbol{\sigma} \int_{-\infty}^{(\delta_f^a - \boldsymbol{\mu})/\boldsymbol{\sigma}} \mathbf{z} \phi(\mathbf{z}) d\mathbf{z} \\ &= \boldsymbol{\mu} - \boldsymbol{\mu} \frac{1 + \eta}{2} + \boldsymbol{\sigma} \int_{-\infty}^{(\delta_f^a - \boldsymbol{\mu})/\boldsymbol{\sigma}} \phi'(\mathbf{z}) d\mathbf{z} \\ &= \boldsymbol{\mu} \left(\frac{1 - \eta}{2}\right) + \boldsymbol{\sigma} \phi\left[(\delta_f^a - \boldsymbol{\mu})/\boldsymbol{\sigma}\right] \\ &= \boldsymbol{\mu} \left(\frac{1 - \eta}{2}\right) + \boldsymbol{\sigma} \phi\left[\Phi^{-1}\left(\frac{1 + \eta}{2}\right)\right]. \end{aligned}$$

With the above Equations the definition of CVaR (as shown in Definition 5), we can have the following safety bounds,

$$\begin{aligned} \text{CVaR}_{\delta_f^a}(\mathbf{X}) &= \frac{\boldsymbol{\mu} \left(\frac{1 - \eta}{2}\right) + \boldsymbol{\sigma} \phi\left[\Phi^{-1}\left(\frac{1 + \eta}{2}\right)\right]}{1 - F(\delta_f^a)} \\ &= \frac{\boldsymbol{\mu} \left(\frac{1 - \eta}{2}\right) + \boldsymbol{\sigma} \phi\left[\Phi^{-1}\left(\frac{1 + \eta}{2}\right)\right]}{\frac{1 - \eta}{2}} \\ &= \boldsymbol{\mu} + \frac{\boldsymbol{\sigma} \phi\left[\Phi^{-1}\left(\frac{1 + \eta}{2}\right)\right]}{\frac{1 - \eta}{2}}. \end{aligned}$$

(B). The second side safety bound via CVaR:

With proof of the first side CVaR safety bound, we can easily have the second side CVaR safety bound, which is shown as follows:

$$\begin{aligned}
\int_{-\infty}^{-\delta_f^b} x f(x) dx &= E[z < (-\delta_f^b - \mu)/\sigma] \\
&= \int_{-\infty}^{(-\delta_f^b - \mu)/\sigma} (\mu + \sigma z) \phi(z) dz \\
&= \int_{-\infty}^{(-\delta_f^b - \mu)/\sigma} \mu \phi(z) dz + \int_{-\infty}^{(-\delta_f^b - \mu)/\sigma} (\sigma z) \phi(z) dz \\
&= \mu \Phi\left[(-\delta_f^b - \mu)/\sigma\right] + \sigma \int_{-\infty}^{(-\delta_f^b - \mu)/\sigma} z \phi(z) dz \\
&= \mu \left(\frac{1-\eta}{2}\right) - \sigma \phi\left[(-\delta_f^b - \mu)/\sigma\right] \\
&= \mu \left(\frac{1-\eta}{2}\right) - \sigma \phi\left[\Phi^{-1}\left(\frac{1-\eta}{2}\right)\right], \\
\text{CVaR}_{-\delta_f^b}(X) &= \frac{\mu \left(\frac{1-\eta}{2}\right) - \sigma \phi\left[\Phi^{-1}\left(\frac{1-\eta}{2}\right)\right]}{F(-\delta_f^b)} \\
&= \frac{\mu \left(\frac{1-\eta}{2}\right) - \sigma \phi\left[\Phi^{-1}\left(\frac{1-\eta}{2}\right)\right]}{\frac{1-\eta}{2}} \\
&= \mu - \frac{\sigma \phi\left[\Phi^{-1}\left(\frac{1-\eta}{2}\right)\right]}{\frac{1-\eta}{2}}.
\end{aligned}$$

□

Thus, we can have CVaR safety bound for equality constraints in terms of safe probability, constraints' mean and constraints' standard deviation, as shown in Figure 6.5.

In the following section, we will introduce a practical algorithm aimed at effectively addressing and managing these stochastic constraints.

6.4.3 Practical Algorithm

Building upon the theoretical foundations outlined earlier, we have devised a practical algorithm, as detailed in Algorithm 7, designed to ensure safety in complex environments characterized by stochastic constraints. The algorithm unfolds in three key steps. Initially, stochastic constraints are formulated within the framework of a manifold. Subsequently, safety bounds are computed through the utilization of VaR and CVaR metrics. Finally, the algorithm projects the safe operational space onto the manifold and orchestrates the search for a secure policy that adheres to the stochastic constraint manifold.

Notably, to simplify computation, we leverage Gaussian noises to represent stochastic constraints $\epsilon_i(\mathbf{q})$, which can be easily learned via Gaussian processes [258]. As shown in Equation (6.27), f represents the stochastic constraints that equal stochastic constraints $\epsilon_i(\mathbf{q})$ plus deterministic constraints $c_i(\mathbf{q})$, and we need to compute f 's safety bounds given $c_i(\mathbf{q})$ and $\epsilon_i(\mathbf{q})$.

$$f = c_i(\mathbf{q}) + \epsilon_i(\mathbf{q}) \quad \epsilon_i(\mathbf{q}) \sim \mathcal{N}(0, \sigma_n^2). \quad (6.27)$$

The stochastic constraints are assumed as independent, and the constraints are subject to independent distributions.

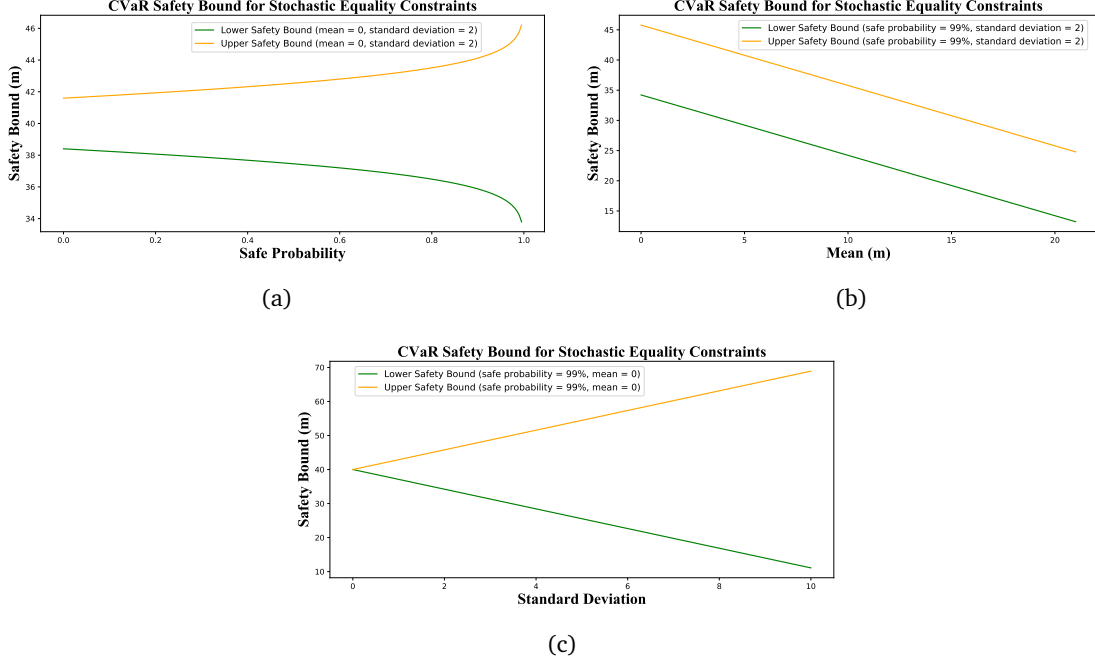


Figure 6.5: CVaR safety bounds on a manifold for equality constraints in terms of safe probability (a), constraints' mean (b), and constraints' standard deviation (c).

Algorithm 7 Searching a **RO**bst **S**afe Policy on a Stochastic **CO**nstraint **M**anifold (**ROSCOM**)

- 1: Input stochastic constraints $\mathbf{c}_i(\mathbf{q}) + \epsilon_i$.
 - 2: Initialise a policy.
 - 3: **for** $k = 0, 1, \dots, T$ **do**
 - 4: Learn stochastic constraints via Gaussian Processes.
 - 5: Compute the stochastic constraints $\mathbf{c}_i(\mathbf{q}) + \epsilon_i$ at each step.
 - 6: Compute the VaR and CVaR bounds for high dimensional inequality and equality constraints.
 - 7: Project the safety bounds into the manifold space $\mathbf{c}(\mathbf{q}, \mu)$.
 - 8: Compute the Jacobian matrix $\mathbf{J}_i(\mathbf{q}, \mu_s) = \left[\frac{\partial}{\partial \mathbf{q}} \bar{\mathbf{J}}_i(\mathbf{q}, \mu_s)^\top, \frac{\partial}{\partial \mu} \bar{\mathbf{J}}_i(\mathbf{q}, \mu_s)^\top \right] \in \mathbb{R}^{(n+N) \times N}$.
 - 9: Compute the null space $\mathbf{N}(\mathbf{q}, \mu_s)$.
 - 10: Sampling trajectories on a tangent space of a constrained manifold with RL algorithms.
 - 11: Compute action $\mathbf{a}_k = \Lambda(\bar{\mathbf{q}}_k)$ based on ATACOM.
 - 12: Deploy action $\mathbf{a}_k$ in the environment and provide the reward and observation to the RL algorithm.
 - 13: **end for**
-

6.5 Experiments

In this section, we conduct a series of experiments with the primary objective of evaluating the effectiveness of our proposed method. Furthermore, we undertake a comparative analysis to assess the performance of our approach in relation to existing SOTA safe RL baselines.

Specifically, we first compare our method with ATACOM [149] on circular motion tasks. Furthermore, to comprehensively evaluate the effectiveness of our methods, we compare our method with ATACOM and traditional CMDP algorithms on challenging air-hockey tasks. Generally, ATACOM requires constraint information, which is leveraged to guarantee safety

most rigorously. For example, the constraint information is human posture in a human-robot interaction environment. However, traditional CMDP algorithms do not require constraint information, and they ensure safety by try-and-error learning, which could be helpful in uncritical safety environments. The representative SOTA algorithms of traditional CMDP settings are CPO [5], PCPO [271], FOCOPS [290], and PPO-Lagrangian [208], and so on.

The environment settings are provided in Appendix 6.7.1 and details of implementation are introduced in Appendix 6.7.2

6.5.1 Circular Motion Tasks

In the circular motion task, a robot must run on the blue circle as shown in Figure 6.6. If the robot runs away from the circle, it will incur a cost. However, the constraint is stochastic. The blue circle denotes the original constraint, the blue points denote the stochastic constraints, and the two green circles denote the safety bounds via VaR, the two red circles represent the safety bounds via CVaR for the new constraints.

6.5.1.1 Compare with ATACOM

The experimental results are presented in Figures 6.7 (a) and (b), where c_{avg} denotes the average cost of each trajectory, J denotes the reward performance, ROSCOM-PPO and ATACOM-PPO [149] denote our ROSCOM with PPO methods [218], and ATACOM with PPO methods. Although the ATACOM method presents remarkable performance for ensuring robot learning safety, a comparative analysis of experimental outcomes suggests that our proposed methodology outperforms ATACOM. More precisely, the proposed method demonstrates superior reward and enhanced safety performance while taking into consideration the stochastic constraints.

6.5.1.2 Compare with Traditional CMDP Algorithms

In Figure 6.8, we present the results of comparison experiments focused on tasks involving circular motion. Specifically, as depicted in Figure 6.8(a), our proposed method not only maintains rigorous safety standards but also significantly outperforms traditional safe RL baselines, including representative baselines CPO [5] and PCPO [271]. These baselines fail to consistently ensure safety during the learning process. Further, Figure 6.8(b) illustrates that our method achieves performance comparable to that of the safe RL baselines. These experimental outcomes demonstrate that our approach consistently surpasses traditional safe RL baselines in safety and reward performance, affirming its efficacy and reliability in safe RL applications.

6.5.2 Air-Hockey Tasks

In the Air-Hockey task, as shown in Figure 6.9, the robot needs to learn to manipulate its hand and hit a lightweight plastic puck to score goals. However, due to noises, e.g., sensor measurement errors, the table height estimation by sensors can vary randomly. To prevent the robot from colliding with the table and causing damage to the robot and the table, we need to compute safety bounds for robot learning, so that the robot can complete the task safely, even with stochastic constraints.

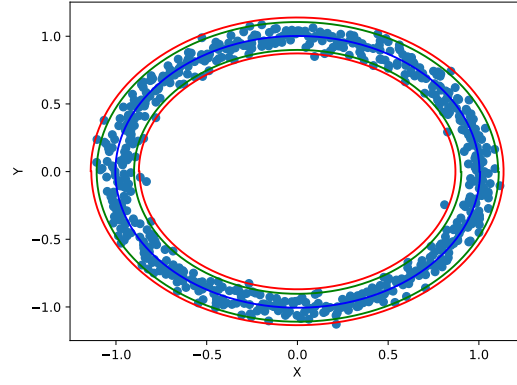


Figure 6.6: Safety bounds on a stochastic constraint manifold. The green and red circles denote the safety bounds via VaR and CVaR. In stochastic constraints with Gaussian parameters, the mean is 0, the standard deviation is 0.05, and the safety probability is 99%.

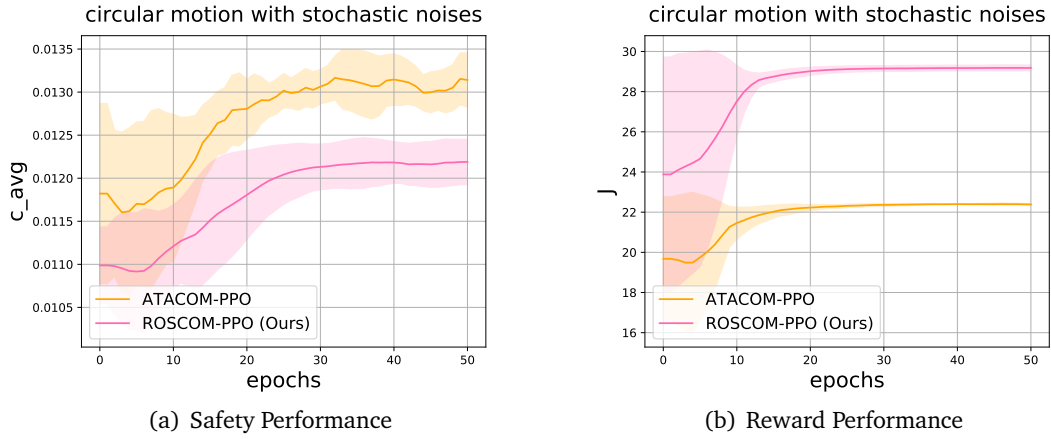


Figure 6.7: Compare our method with ATACOM [149] on a circular motion task regarding safety (lower values signify superior performance) and reward (higher values indicate better performance) performance.

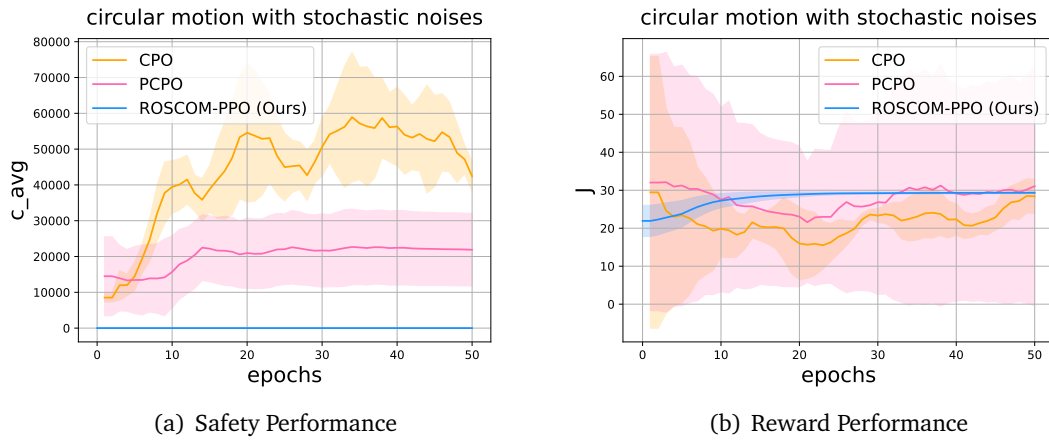


Figure 6.8: Compare our method with traditional CMDP methods, CPO [5] and PCPO [271], on a circular motion task regarding safety and reward performance.

6.5.2.1 Compare with ATACOM

As depicted in Figure 6.10, we compare our method with ATACOM, the experimental results demonstrate that our method is better than ATACOM in terms of safety and reward performance, e.g., the average cost value of trajectories is lower than ATACOM (Figure 6.10 (a)), and reward value is higher than ATACOM (Figure 6.10 (b)). The results indicate that our method can be more rigorous than ATACOM to ensure safety for critical applications.

6.5.2.2 Compare with Traditional CMDP Algorithms

In order to provide a comprehensive evaluation of the efficacy of our proposed method, we extend our assessment by conducting comparative experiments. Specifically, we compare our method with SOTA representative safe RL baselines, including CPO [5] and PCPO [271].

The evaluation results, as depicted in Figure 6.11, reveal the remarkable superiority of our method in comparison to SOTA baselines regarding safety and reward performance. Firstly, our method exhibits significantly improved safety performance, as evidenced by the average cost values of each trajectory, as illustrated in Figure 6.11 (a). Secondly, concerning reward performance, our approach outperforms the strong baselines, as demonstrated by Figures 6.11 (b) and (c), with Figure 6.11 (c) providing an enlarged view of Figure 6.11 (b) in terms of SOTA baselines' reward performance. Our methodology's capability to attain superior outcomes is unequivocally demonstrated, establishing its preeminence over the strong SOTA baselines and affirming its potential for robust and safe RL applications.

The distinct superiority of our method over traditional algorithms can be attributed to several key elements integrated within our approach. Firstly, the constrained space is leveraged to construct a constrained manifold, a feature that proves to be instrumental in facilitating efficient high-dimensional learning. Secondly, the tangent space of the constrained manifold is explicitly established for policy searching. It instills a rigorous safety assurance mechanism within the policy search process. By operating within the confines of this specially designed tangent space, the policy search is inherently aligned with safety protocols, ensuring that each iteration adheres to predetermined safety constraints.

These key points collectively contribute to the marked performance elevation of our method, positioning it as a notably advanced solution compared to traditional algorithms, and promising enhanced safety and efficiency in complex, high-dimensional learning environments.

Influence of Safety Bounds on Reward Performance: (1) In our experiments, the safety bound significantly reduces learning oscillations during policy search, enhancing overall policy performance. For example, in the circular motion task, the state space's stochastic noise can lead to policy oscillations in the baseline algorithm such as ATACOM, subsequently degrading policy performance. Similar effects are observed in air hockey tasks. Unlike these methods, our approach establishes a stable safety bound that accommodates the uncertainty of the state space. This allows for more effective policy exploration than baseline methods, thereby improving safety and reward outcomes. (2) Compared to traditional safe RL methods, our approach is developed on constrained manifolds. This design facilitates safe exploration within the tangent space and guides the policy toward a higher-quality outcome. Consequently, our method achieves superior safety and reward performance relative to traditional approaches.

6.6 Summary of Safe Robust Reinforcement Learning

In this paper, we investigated the problem of robust safe RL on stochastic constraint manifolds. Prior safe RL research has focused on deterministic constraints, whereas in this paper we considered stochastic constraints. First, we formulated the robust safe RL problem by considering

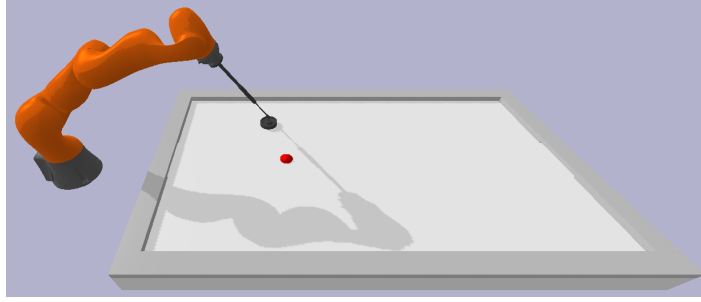


Figure 6.9: An Air-Hockey task environment.

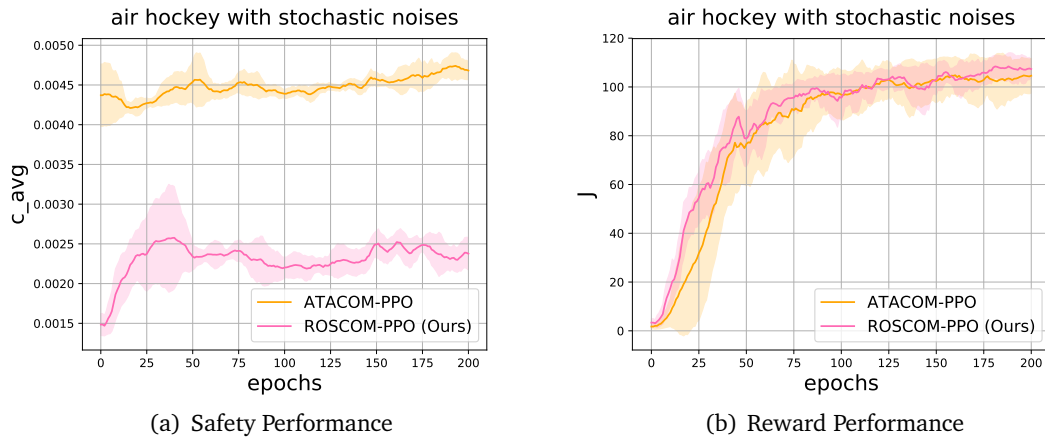


Figure 6.10: Compare our method with ATACOM [149] on the Air-Hockey task.

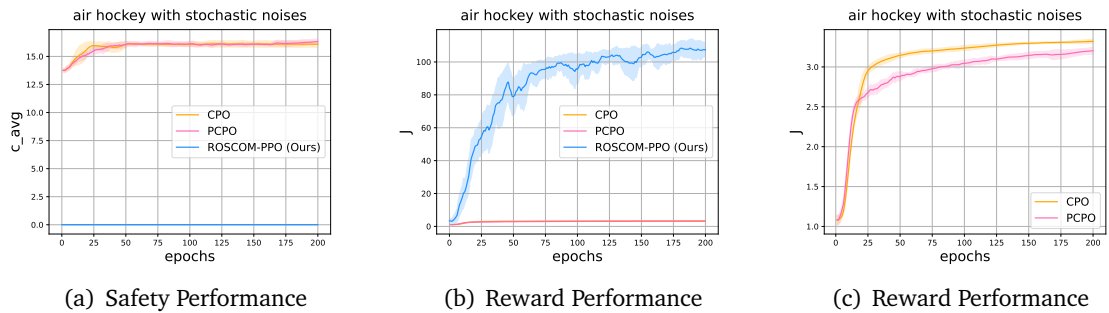


Figure 6.11: Compare our method with traditional CMDP methods, CPO [5] and PCPO [271], on the Air-Hockey task.

the stochastic constraints. Second, we ensured safety by leveraging the risk measurement methods, e.g., VaR and CVaR. Safety bounds are computed on the stochastic constraint manifold to help search for a safe policy. Finally, we evaluated our method on the circular motion and Air-Hockey tasks. The experiment results demonstrate that our algorithm can achieve remarkable performance in terms of learning safety, and show better reward performance than the SOTA baselines. In the Future, we plan to investigate the model's efficiency and try to deploy our method in real-world robot control.

6.7 Details of Safe Robust Reinforcement Learning Experiments

6.7.1 Environment Settings of Safe Robust Reinforcement Learning

Circle Motion Environments. In this task, the robot needs to run as fast as possible to reach the goal position while satisfying safety constraints. As shown in the following equations, $R(X_t, Y_t)$ denotes the reward value when the robot is at position (X_t, Y_t) , X_t denotes the robot X-axis direction position and Y_t denotes its Y-axis direction position, similarly, (X_g, Y_g) denotes the goal position, ϵ denotes the stochastic constraints, and $C_{inequality}$ denotes the inequality constraints. other settings are similar to ATACOM [149].

$$R(X_t, Y_t) = \exp(-\sqrt{(X_t - X_g)^2 + (Y_t - Y_g)^2}) \quad (6.28)$$

$$C(X_t, Y_t) = |\sqrt{X_t^2 + Y_t^2} - (1 + \epsilon)| + C_{inequality} \quad (6.29)$$

$$C_{inequality} = \begin{cases} |Y_t + 0.5 + \epsilon|, & Y_t < -(0.5 + \epsilon), \\ 0, & \text{Others.} \end{cases} \quad (6.30)$$

Air-Hockey Environments.

Other reward and constraint settings of the task are same to ATACOM [149], except for Equation (6.31), Z_{table} denotes the table height, Z_t denotes the robot Z-axis direction position, *puck* height is 0.0149m. We provide more experiments to evaluate the effectiveness of our method regarding the different safety limits, as shown in Figure 6.12, the experiment results indicate that our method can perform remarkably better than SOTA safe RL baselines regarding reward and safety performance, and confirm the effectiveness of our method again. The implementation of CPO, PCPO, FOCOPS and PPO-Lag from a repository of safe RL baselines¹ is used to carry out the comparison experiments.

Our method outperforms the SOTA safe RL baselines because the safety constraints are learned and projected on a Manifold space, which means our algorithm can receive the safety and task environment information in advance, which could help our method better search policy while satisfying safety constraints.

$$C_{inequality} = \begin{cases} |Z_t - (Z_{table} + \epsilon)|, & Z_t < (Z_{table} + \epsilon), \\ 0, & \text{Others.} \end{cases} \quad (6.31)$$

6.7.2 Details of Safe Robust Reinforcement Learning Experiments

The parameters used in our experiments are provided in Tables 6.1 and 6.2.

¹<https://github.com/PKU-Alignment/Safe-Policy-Optimization.git>

Parameters	value	Parameters	value
actor lr	3e-4	critic lr	3e-4
batch size	64	gamma	0.99
horizon	500	epoch	50
eps ppo	0.1	noise ϵ mean	0
η	99%	noise ϵ std	0.5
network	MLP	regular	relu & linear
NN features	[32, 32]		

Table 6.1: ROSCOM and ATACOM hyparameters used in Circle Motion experiments.

Parameters	value	Parameters	value
actor lr	3e-4	critic lr	3e-4
batch size	64	gamma	0.99
horizon	3000	epoch	200
eps ppo	0.1	noise ϵ mean	0
η	99%	noise ϵ std	0.5
network	MLP	regular	relu & linear
NN features	[64, 64]		

Table 6.2: Algorithms' hyparameters of ROSCOM, ATACOM, CPO, PCPO, FOCOPS, and PPO-Lag used in Air-Hockey experiments. Safety bound δ used for CPO, PCPO, FOCOPS, and PPO-Lag in Figure 6.11 is 25 and in Figure 6.12 is 5.

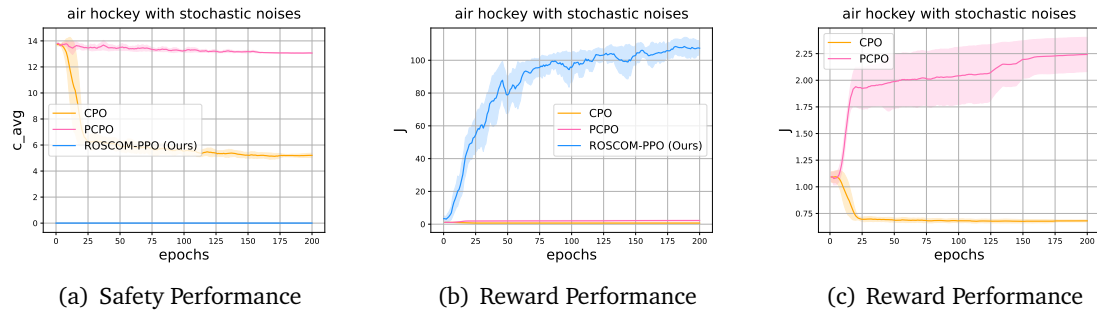


Figure 6.12: Compare our method with traditional CMDP methods, CPO [5] and PCPO [271], on the Air-Hockey task with different safety bounds.

7

Safe Robot Reinforcement Learning Benchmarks

Within the scope of this dissertation, we introduce a comprehensive framework for the assessment of safe robot RL methods, comprising three categories of benchmarks. These benchmarks serve as instrumental tools for evaluating the performance of safe robot RL methodologies across the domains of single-robot RL, multi-objective RL, and multi-robot RL, ensuring a thorough and fair evaluation process. The benchmarks for safe single robot RL are detailed in Section 7.1 of this dissertation. Further, the benchmarks pertaining to safe multi-objective robot RL are introduced in Section 7.2. Lastly, the benchmarks focused on safe multi-robot RL are provided in Section 7.3. This chapter provides a comprehensive overview and analysis of the respective benchmarks, offering valuable insights into their applications in the field of safe robot RL.

7.1 Safe Single Robot Reinforcement Learning Benchmarks

We have designed a *Safety-MuJoCo Benchmark* based on MuJoCo [244] to evaluate the performance of safe RL algorithms. This benchmark differs from traditional safe RL benchmarks, such as *Omnisafe*¹ [113] that is designed based on Safety Gym² [208]. In most of *Omnisafe*'s MuJoCo tasks, the cost constraint is set as the velocity limit, and the reward is determined by the speed at which the robot runs. Conversely, our benchmark considers not only velocity constraints but also the health of the robot. For example, whether the robot falls or whether its joints exceed the limit values of motion control are taken into account.

SafetyWalker-v4. As depicted in Figure 7.1 (a), the reward setting for the SafetyWalker-v4 task is illustrated in Equation (7.1). In this equation, X_v represents the forward velocity of the robot, and α_r signifies the reward weight. A higher reward is achieved as the robot's speed increases. The cost settings can be observed in Equation (7.2), in which α_c denotes the cost weight and a_i corresponds to the values of the i^{th} action. A smaller cost value indicates that the robot conserves more energy.

$$R_{safetywalker} = \alpha_r X_v. \quad (7.1)$$

¹<https://github.com/PKU-Alignment/omnisafe.git>

²<https://github.com/openai/safety-gym.git>

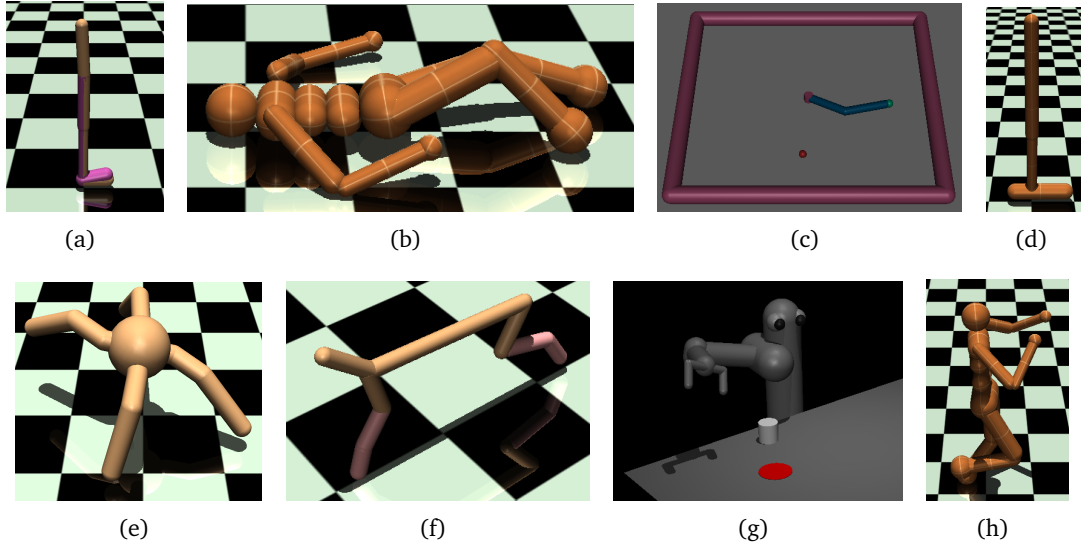


Figure 7.1: Safety-MuJoCo Environments: SafetyWalker-4 (a), SafetyHumanoidStandup-v4 (b), SafetyReacher-v4 (c), SafetyHopper-v4 (d), SafetyAnt-v4 (e), SafeHalfCheetah-v4 (f), SafetyPusher-v4 (g) and SafetyHumanoid-v4 (h).

$$C_{safetywalker} = \alpha_c \sum_{j=0}^J a_i^2. \quad (7.2)$$

SafetyHumanoidStandup-v4. As presented in Figure 7.1 (b), the reward settings for the SafetyHumanoidStandup task can be observed in Equation (7.3), where Z_h denotes the height of the robot standing up. A higher value for the robot's height results in an increased reward value. Additionally, the cost settings are demonstrated in Equation (7.4), in which a_i corresponds to the values of the i^{th} action, and $f_{k,force}$ represents the force values of the k^{th} mass.

$$R_{safetyhumanoidstandup} = Z_h. \quad (7.3)$$

$$C_{safetyhumanoidstandup} = 0.1 * (\sum_j a_i)^2 + 0.5e - 6 * (\sum_{k=0}^K f_{k,force})^2. \quad (7.4)$$

SafetyReacher-v4. As depicted in Figure 7.1 (c), the reward settings for the SafetyReacher task are presented in Equation (7.5). Here, d_{ft} represents the distance between the robot's fingers and the goal position, with a smaller distance yielding a higher reward. Moreover, the cost settings can be observed in Equation (7.6), where, similarly, a_i corresponds to the values of the i^{th} action.

$$R_{safetyreacher} = -\|d_{ft}\|_2. \quad (7.5)$$

$$C_{safetyreacher} = \sum_{j=0}^J a_i^2. \quad (7.6)$$

SafetyHopper-v4. As depicted in Figure 7.1 (c), the reward settings for the SafetyHopper task can be observed in Equation (7.7). These settings are similar to those of SafetyWalker, indicating that the reward value increases as the robot's speed accelerates. Furthermore, the cost settings are presented in Equation (7.8). The first part of the cost settings resembles those of SafetyWalker, while the second part pertains to the robot's health. If the robot satisfies the health conditions, no punishment is incurred; however, if it fails to meet these conditions, a cost value of 1 is applied.

$$R_{safetyhopper} = \alpha_r X_v. \quad (7.7)$$

$$C_{safetyhopper} = \alpha_c \sum_{j=0}^J a_i^2 + C_{he}. \quad (7.8)$$

$$C_{he} = \begin{cases} 0, & 3 = R_{he-state} + R_z + R_{angle}, \\ 1, & \text{Others.} \end{cases} \quad (7.9)$$

$$R_{he-state} = \begin{cases} 1, & State_{min} \leq State_{real} \leq State_{max}, \\ 0, & \text{Others.} \end{cases} \quad (7.10)$$

$$R_z = \begin{cases} 1, & Z_{min} \leq Z_{real} \leq Z_{max}, \\ 0, & \text{Others.} \end{cases} \quad (7.11)$$

$$R_{angle} = \begin{cases} 1, & Angle_{min} \leq Angle_{real} \leq Angle_{max}, \\ 0, & \text{Others.} \end{cases} \quad (7.12)$$

SafetyAnt-v4. As illustrated in Figure 7.1 (e), the reward settings for the SafetyAnt task are presented in Equation (7.13). These settings resemble those of SafetyWalker, signifying that a higher reward is achieved as the robot's speed increases. Additionally, the cost settings are similar to those of SafetyWalker. A smaller cost value indicates that the robot consumes less energy.

$$R_{safetyant} = X_v. \quad (7.13)$$

$$C_{safetyant} = \alpha_c \sum_{j=0}^J a_i^2. \quad (7.14)$$

SafetyHalfCheetah-v4. As shown in Figure 7.1 (f), the reward settings for the SafetyHalfCheetah task are presented in Equation (7.15). In this equation, V_{target} denotes the robot's target velocity, and $V_{cheetah}$ represents the robot's current velocity. A higher reward value is achieved as the current speed approaches the target speed. The cost settings can be observed in Equation (7.16), where $H_{cheetah}$ denotes the current height of the robot, and H_{target} represents the constrained height of the robot. If the current speed exceeds the target speed by a greater margin, more cost will be incurred.

$$R_{safetymhalfcheetah} = -|V_{cheetah} - V_{target}|. \quad (7.15)$$

$$C_{safetymhalfcheetah} = |H_{cheetah} - H_{target}|. \quad (7.16)$$

SafetyPusher-v4. As depicted in Figure 7.1 (g), the reward settings for the SafetyPusher task can be observed in Equation (7.17). Here, $D_{object-goal}$ represents the distance between the object and the goal position, while $D_{robot-object}$ denotes the distance between the robot and the object. A higher reward is achieved when the values of these two distances are smaller. The cost settings for SafetyPusher are similar to those of the SafetyWalker task.

$$R_{safetypusher} = -\|D_{object-goal}\|_2 - \|D_{robot-object}\|_2. \quad (7.17)$$

$$C_{safetypusher} = \alpha_c \sum_{j=0}^J a_i^2. \quad (7.18)$$

SafetyHumanoid-v4. As depicted in Figure 7.1 (g), the reward settings for the SafetyHumanoid task are similar to those of SafetyWalker. The first part of the cost settings also shares similarities with SafetyWalker. The second part of the cost settings is related to the robot's state of health, specifically concerning the robot's standing distance. If the standing distance satisfies certain conditions, no punishment is incurred; however, if it does not meet these conditions, a cost of 1 is emitted.

$$R_{safetymhumanoid} = \alpha_r X_v. \quad (7.19)$$

$$C_{safetymhumanoid} = \alpha_c \sum_{j=0}^J a_i^2 + C_z. \quad (7.20)$$

$$C_z = \begin{cases} 0, & Z_{min} \leq Z_{real} \leq Z_{max}, \\ 1, & Others. \end{cases} \quad (7.21)$$

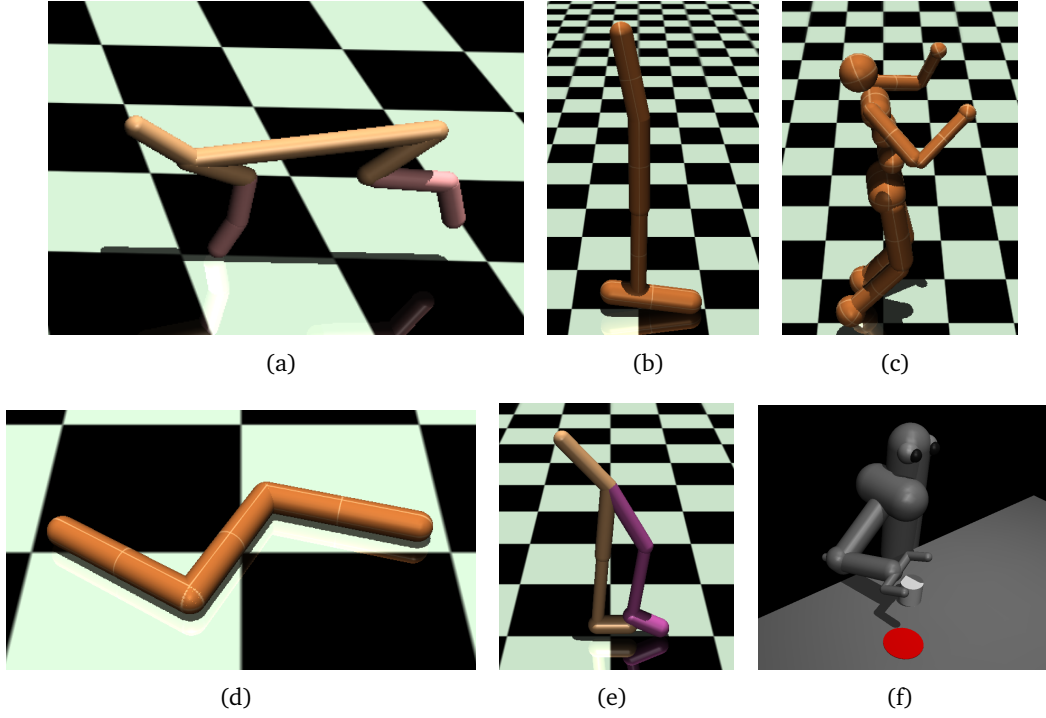


Figure 7.2: Safe Multi-Objective MuJoCo Environments. Specifically, these environments are Safe Multi-Objective HalfCheetah (a), Safe Multi-Objective Hopper (b), Safe Multi-Objective Humanoid (c), Safe Multi-Objective Swimmer (d), Safe Multi-Objective Walker (e) and Safe Multi-Objective Pusher (f).

7.2 Safe Multi-Objective Robot Reinforcement Learning Benchmarks

Similar to *Safety-MuJoCo*, several Safe Multi-Objective environments³ have been designed to evaluate the performance of safe MORL algorithms by leveraging MuJoCo [39, 244]. As illustrated in Figure 7.2, Safe Multi-Objective HalfCheetah (Figure 7.2 (a)), Safe Multi-Objective Hopper (Figure 7.2 (b)), Safe Multi-Objective Humanoid (Figure 7.2 (c)), Safe Multi-Objective Swimmer (Figure 7.2 (d)), Safe Multi-Objective Walker (Figure 7.2 (e)), and Safe Multi-Objective Pusher (Figure 7.2 (f)) have been introduced to evaluate the effectiveness of our proposed methods.

Safe Multi-Objective HalfCheetah. As demonstrated in Figure 7.2 (a), the constraint for the Safe Multi-Objective HalfCheetah is the difference between the robot’s real-time head height and its desired head height. This is represented in Equation (7.22), where C_h^i denotes the cost value at each step, $H_{cheetah}^i$ represents the real-time head height of the robot, and H_{target}^i indicates the target height. We define two reward functions for the robot’s tasks.

The first reward function is energy conservation, which requires the robot to optimize its actions to minimize energy consumption while achieving its goal. In Equation (7.24), R_e^i denotes the energy of action $a_{cheetah}^i$ at each step i , and α_a represents the energy weight.

The second reward function is related to velocity, where the robot must learn to run as quickly as possible to achieve the velocity goal. As shown in Equation (7.23), R_v^i represents the velocity reward at each step, while $V_{cheetah}^i$ and V_{target}^i denote the real-time velocity and target velocity at each step, respectively.

³<https://github.com/SafeRL-Lab/Safe-Multi-Objective-MuJoCo.git>

$$C_h^i = |H_{cheetah}^i - H_{target}^i|. \quad (7.22)$$

$$R_v^i = -|V_{cheetah}^i - V_{target}^i|. \quad (7.23)$$

$$R_e^i = -\alpha_a \|a_{cheetah}^i\|^2. \quad (7.24)$$

Safe Multi-Objective Hopper. As illustrated in Figure 7.2 (b), the constraint for the Safe Multi-Objective Hopper is the energy computed using the action values. In Equation (7.25), $C_{e-hopper}^i$ denotes the cost value at each step. There are two objectives in this setting.

The first reward function is the forward reward function, as shown in Equation (7.26). The forward reward function, R_f^i , is weighted by α_v , and X_v represents the forward velocity.

The second reward function is the healthy reward, which considers both the robot's angle and state. In Equation (7.27), R_{he}^i denotes the healthy reward, $R_{he-state}$ represents the healthy state (meaning the robot's state must satisfy certain requirements), R_z is the Z-axis robot health, and R_{angle} is the robot angle health. The robot angle must meet the minimum and maximum angle requirements.

Additionally, we modify the reward settings to accommodate three objective tasks, named **Safe Multi-Objective Hopper-v2**. In this setting, R_f^i , $(R_{he-state} + R_z)$, and R_{angle} serve as the three objective rewards, respectively.

$$C_{e-hopper}^i = -\|a_{hopper}^i\|. \quad (7.25)$$

$$R_f^i = \alpha_v X_v. \quad (7.26)$$

$$R_{he}^i = R_{he-state} + R_z + R_{angle}. \quad (7.27)$$

$$R_{he-state} = \begin{cases} 1, & State_{min} \leq State_{real} \leq State_{max}, \\ 0, & Others. \end{cases} \quad (7.28)$$

$$R_z = \begin{cases} 1, & Z_{min} \leq Z_{real} \leq Z_{max}, \\ 0, & Others. \end{cases} \quad (7.29)$$

$$R_{angle} = \begin{cases} 1, & Angle_{min} \leq Angle_{real} \leq Angle_{max}, \\ 0, & Others. \end{cases} \quad (7.30)$$

Safe Multi-Objective Humanoid. As displayed in Figure 7.2 (c), the constraint for the Safe Multi-Objective Humanoid is the control cost value related to energy conservation. In Equation (7.31), $C_{e-humanoid}^i$ represents the cost value at each step i .

The first reward function is the forward reward concerning velocity, similar to Equation (7.26). Higher velocity results in a better forward reward.

The second reward function is the healthy reward, which is associated with the robot's stance. Consequently, a higher height in the Z-axis direction leads to a better healthy reward.

As shown in Equation (7.32), R_{he}^i denotes the healthy reward value, and H_z^i represents the height in the X-axis direction.

$$C_{e-humanoid}^i = -\|a_{humanoid}^i\|^2. \quad (7.31)$$

$$R_{he}^i = H_z^i. \quad (7.32)$$

Safe Multi-Objective Swimmer. As depicted in Figure 7.2 (d), the constraint for the Safe Multi-Objective Swimmer is related to action energy. In Equation 7.33, $C_{e-swimmer}^i$ represents the cost value at each step, $\alpha_{swimmer}$ denotes the cost weight, and $a_{swimmer}^i$ indicates the action value. There are two reward functions in this environment.

The first reward function is the move forward reward, which is associated with the X-axis direction velocity. As shown in Equation (7.34), R_{x-v}^i denotes the Y-axis direction reward at each step i , α_v represents the weight of the X-axis velocity, and X_v indicates the X-axis velocity.

The second reward function is the move left reward, which is related to the Y-axis direction velocity. In Equation (7.35), R_{y-v}^i denotes the Y-axis direction reward at each step i , and Y_v represents the Y-axis velocity. Higher velocities lead to better rewards.

$$C_{e-swimmer}^i = -\alpha_{swimmer} \|a_{swimmer}^i\|. \quad (7.33)$$

$$R_{x-v}^i = \alpha_v X_v. \quad (7.34)$$

$$R_{y-v}^i = Y_v. \quad (7.35)$$

Safe Multi-Objective Walker. As illustrated in Figure 7.2 (e), the constraint for Safe Multi-Objective Walker is related to action energy, as shown in Equation (7.36). There are two reward functions in this environment.

The first reward function is the move forward reward, which is similar to Equation (7.34).

The second reward function is about the healthy reward, which considers healthy angle and healthy Z-axis height. This can be seen in Equation (7.37). The settings for R_z and R_{angle} are similar to those in Equation (7.29) and Equation (7.30).

$$C_{e-walker}^i = -\|a_{walker}^i\|. \quad (7.36)$$

$$R_{he}^i = R_z + R_{angle}. \quad (7.37)$$

Safe Multi-Objective Pusher. As displayed in Figure 7.2 (f), the constraint for the Safe Multi-Objective Pusher is related to action energy. In Equation (7.38), $C_{e-pusher}^i$ represents the cost value at each step i , and a_{pusher}^i is the set of actions at each step i . There are two reward functions in this environment.

The first reward function is concerned with moving the target object towards the goal position. As shown in Equation (7.39), $R_{goal-pusher}^i$ denotes the first reward, and $D_{object-goal}^i$ represents the distance between the target object and the goal position at each step i .

The second reward function is about the robot's end-effector position relative to the target object's position. In Equation (7.40), $R_{object-pusher}^i$ denotes the second reward at each step i , and $D_{robot-object}^i$ indicates the distance between the robot and the object at each step i .

$$C_{e-pusher}^i = \|a_{pusher}^i\|^2. \quad (7.38)$$

$$R_{goal-pusher}^i = -\|D_{object-goal}^i\|. \quad (7.39)$$

$$R_{object-pusher}^i = -\|D_{robot-object}^i\|. \quad (7.40)$$

7.3 Safe Multi-Robot Reinforcement Learning Benchmarks

To evaluate the effectiveness of safe MARL methods, three safe multi-agent benchmarks have been designed by [89], namely Safe Multi-Agent MuJoCo (Safe MAMuJoCo), Safe Multi-Agent Robosuite (Safe MARobosuite), Safe Multi-Agent Isaac Gym (Safe MAIG), respectively. The safe multi-agent benchmarks can help promote the research of safe MARL.

7.3.1 Safe Multi-Agent MuJoCo

Safe MAMuJoCo [89]⁴ is an extension of MAMuJoCo [195]. In Safe MAMuJoCo, safety-aware agents have to learn not only the skillful manipulations of a robot, but also how to avoid crashing into unsafe obstacles and positions. In particular, the background environment, agents, physics simulator, and reward function are preserved. However, unlike its predecessor, a Safe MAMuJoCo environment comes with obstacles like walls or bombs. Furthermore, with the increasing risk of an agent stumbling upon an obstacle, the environment emits cost [39]. According to the scheme in [280], we characterize the cost functions for each task.

For visualization, refer to Figure 7.3. Specifically, we evaluate safe multi-agent reinforcement learning (MARL) on the Ant and ManyAgent Ant tasks, focusing on both reward and safety performance, and assessing how safely the robots can navigate cooperatively in static obstacle environments. The Safe ManyAgent Ant, Humanoid, and Ant tasks are designed for static obstacles, while the Safe HalfCheetah and Hopper tasks are set in dynamic obstacle environments. Following the approach outlined in [280], we characterize the cost functions for each task below.

ManyAgent Ant Task 1.0 & Ant Task 1.0

The width of the corridor set by two walls is 9 m (ManyAgent Ant with 2 agents (2x3), 3 agents (3x2), 6 agents (6x1))/10 m (Ant with 2 (2x4, 2x4d), 4 (4x2), 8 (8x1) agents). The environment emits the cost of 1 for an agent, if the distance between the robot and the wall is less than 1.8 m, or when the robot topples over, see Figure 7.4 (a) and (b). This can be described as

$$c_t = \begin{cases} 0, & \text{for } 0.2 \leq z_{torso,t+1} \leq 1.0 \text{ and } z_{rot} > -0.7 \\ & \text{and } \|\mathbf{x}_{torso,t+1} - \mathbf{x}_{wall}\|_2 \geq 1.8, \\ 1, & \text{otherwise,} \end{cases}$$

⁴<https://github.com/chauncygu/Safe-Multi-Agent-Mujoco.git>

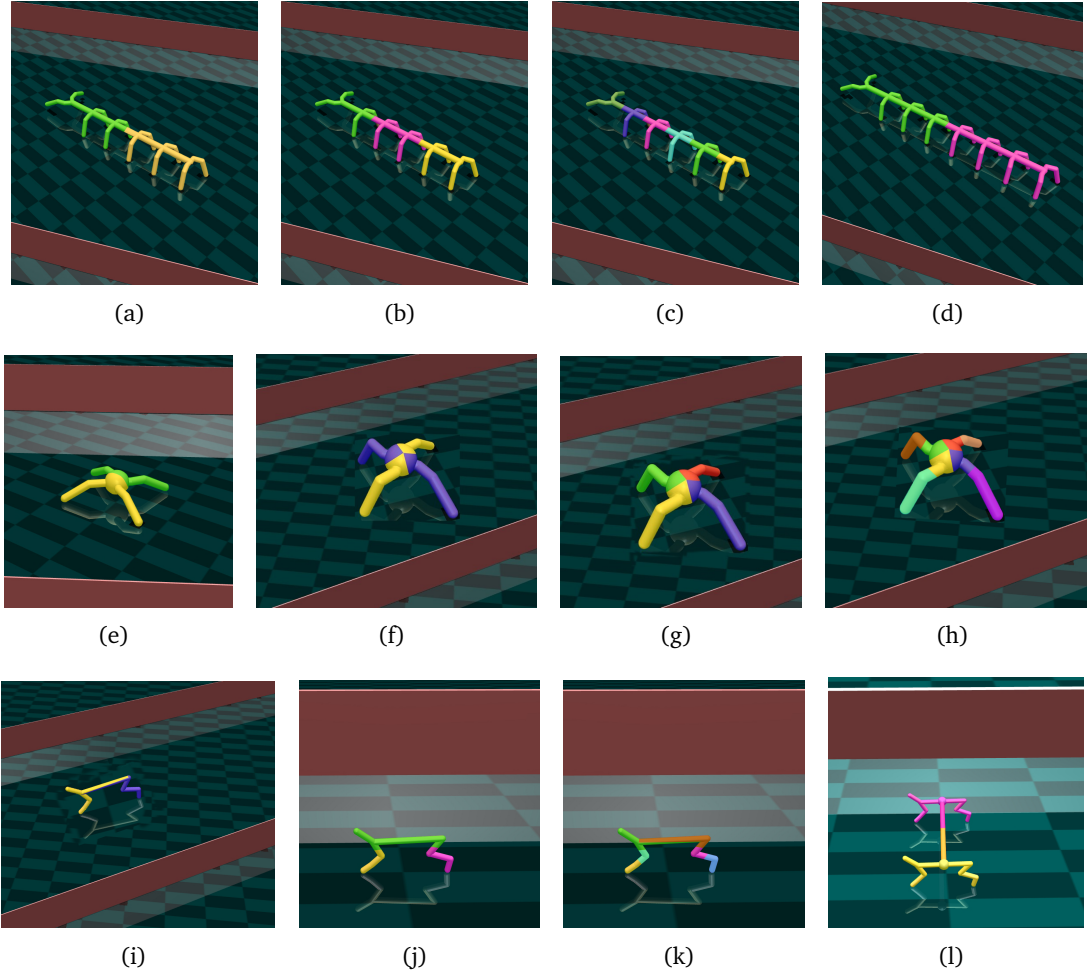


Figure 7.3: Example tasks in Safe Multi-Agent MuJoCo. (a)-(d): Safe ManyAgent Ant, (e)-(h): Safe Ant, (i)-(l): Safe HalfCheetah. Body parts of different colours are controlled by different agents. Agents jointly learn to manipulate the robot, while avoiding crashing into unsafe red areas.

where $\mathbf{z}_{\text{torso},t+1}$ is the robot's torso's z -coordinate, $\mathbf{z}_{\text{rot}}$ is the robot's rotation's z -coordinate, and $\mathbf{x}_{\text{torso},t+1}$ is the robot's torso's x -coordinate, at time $t + 1$; $\mathbf{x}_{\text{wall}}$ is the x -coordinate of the wall. A similar cost scheme is implemented in *ManyAgent Ant Tasks 2.1 & 2.2* and in *Ant Tasks 2.1 & 2.2*.

HalfCheetah Task 1.0 & Couple HalfCheetah Task 1.0

In these tasks, the HalfCheetah agents move inside a corridor (which constraints their movement, but does not induce costs). Together with them, there are pitfalls moving inside the corridor. If an agent finds itself too close to a bomb, the distance between an agent and a bomb is less than 9 m , a cost of 1 will be emitted, see Figure 7.4 (c) and (d).

$$c_t = \begin{cases} 0, & \text{for } \|\mathbf{y}_{\text{torso},t+1} - \mathbf{y}_{\text{obstacle}}\|_2 \geq 9, \\ 1, & \text{otherwise,} \end{cases}$$

where $\mathbf{y}_{\text{torso},t+1}$ is the y -coordinate of the robot's torso, and $\mathbf{y}_{\text{obstacle}}$ is the y -coordinate of the moving obstacle.

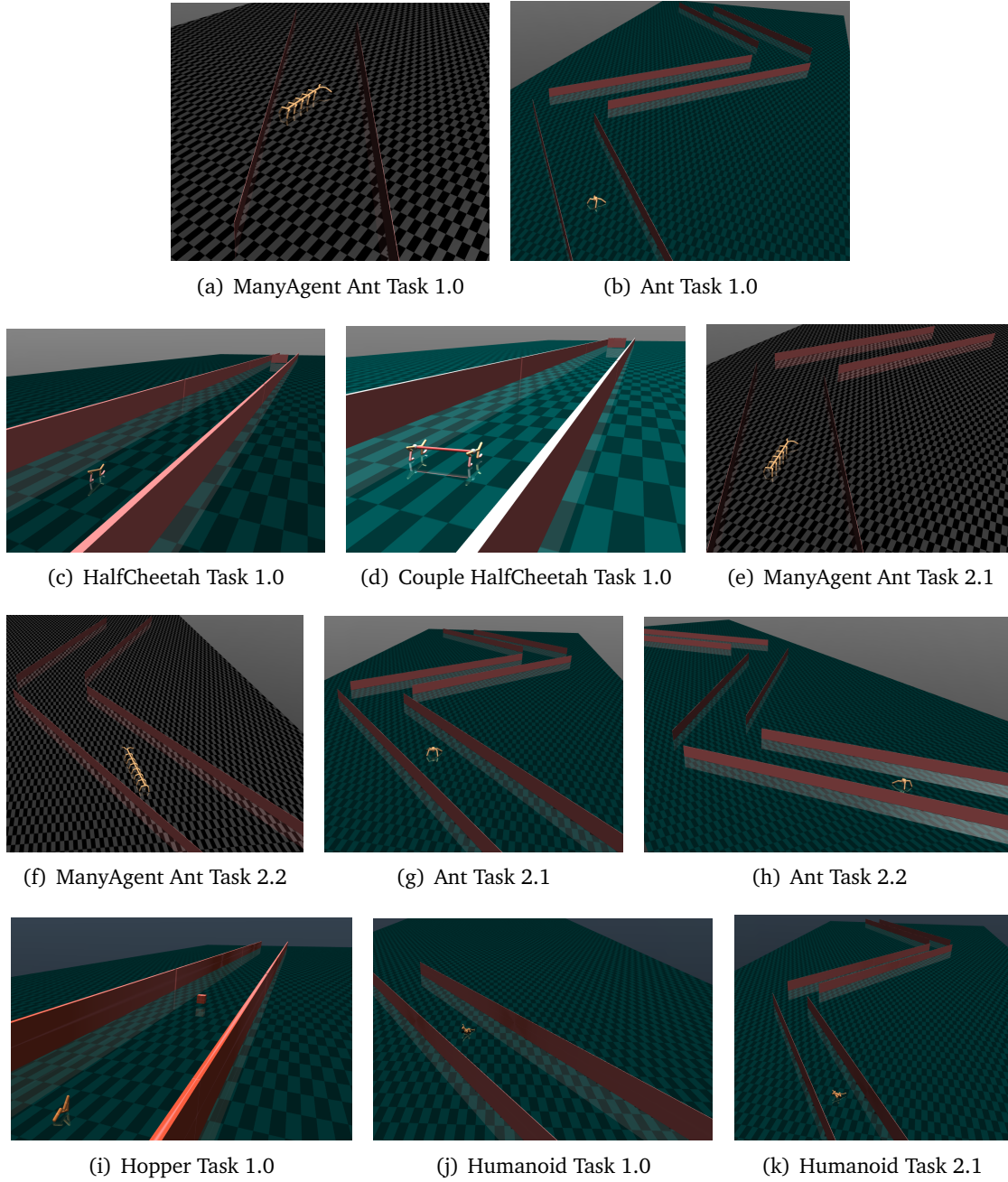


Figure 7.4: Specific tasks in Safe Multi-Agent MuJoCo Environment. (a): ManyAgent Ant Task 1.0: ManyAgent Ant 3x2 one folding line walls (corridor width is 9 m). (b): Ant Task 1.0: Ant 4x2 with three folding Jagged (30°) line walls. (c): HalfCheetah Task 1.0: HalfCheetah 2x3 with moving obstacle, (d): Couple HalfCheetah Task 1.0: Couple HalfCheetah 1P1 with moving obstacle, (e): ManyAgent Ant Task 2.1: Many-Agent Ant 3x2 with two folding line walls (corridor width is 12 m). (f): ManyAgent Ant Task 2.2: Many-Agent Ant 4x2 with two folding line walls (corridor width is 12 m). (g): Ant Task 2.1: Ant 4x2 with three folding Jagged (40°) line walls (corridor width is 10 m). (h): Ant Task 2.2: Ant 4x2 with three folding Jagged (40°) line walls (corridor width is 8 m). (i): Hopper Task 1.0: Hopper 3x1 with moving obstacle. (j): Humanoid task 1.0: Humanoid 17x1 with one folding line walls (corridor width is 7 m). (k): Humanoid task 1.0: Humanoid 17x1 with three folding line walls (corridor width is 9 m).

Hopper 1.0

The Hopper (3x1) robot is in a corridor, and there is a moving obstacle in the corridor (which can be regarded as a pitfall). When the robot encounters the moving obstacle, it will suffer a

cost,

$$c_t = \begin{cases} 0, & \text{for } \|y_{\text{torso},t+1} - y_{\text{obstacle}}\|_2 \geq 1, \\ 1, & \text{otherwise.} \end{cases}$$

Humaniod task 1.0 & Humaniod task 2.1

The Humaniod (17x1) robot run in a corridor, the width of the corridor set by two walls is 7 m(Humaniod task 1.0)/9 m (Humaniod task 2.1). The environment emits the cost of 1 for an agent, if the distance between the robot and the wall is less than 1.8 m, or when the robot topples over, this can be described as

$$c_t = \begin{cases} 0, & \text{for } 1.0 \leq z_{\text{torso},t+1} \leq 2.0 \text{ and } z_{\text{rot}} > -0.7 \\ & \text{and } \|x_{\text{torso},t+1} - x_{\text{wall}}\|_2 \geq 1.8, \\ 1, & \text{otherwise.} \end{cases}$$

7.3.2 Safe Multi-Agent Robosuite

Safe Multi-Agent Robotsuite (Safe MARobosuite) [89]⁵, shown in Figure 7.5, has been designed on the basis of Robosuite [295] which is a popular robotic arm benchmark for single-agent reinforcement learning. In Safe MARobosuite, multiple agents are set up according to the robot joint settings, and each agent controls every joint or several joints. A Lift task, for example, can be divided up into 2 agents (2x4 Lift), 4 agents (4x2 Lift), 8 agents (8x1 Lift); for Stack tasks, similarly, tasks can be provided with 2 agents (2x4 Stack), 4 agents (4x2 Stack), 8 agents (8x1 Stack); for a two-robot TwoArmLift task, it can be divided up into: 2 agents (2x8 TwoArmLift), 4 agents (4x4 TwoArmLift), 8 agents (8x2 TwoArmLift), 16 agents (16x1 TwoArmLift).

Moreover, Safe MARobosuite is also a fully cooperative, continuous, and decentralized benchmark that considers the constraints of robot safety. Where Franka robots are used to conduct each task, each agent can observe partial environmental information (such as the velocity and position). More importantly, Safe MARobosuite can be easily used for modular robots and make robots have good robustness and scalability. In many real-world applications, modular robots can be quickly paired and assembled for different tasks [9]. For instance, when communication bandwidth is limited, or some joints of robotic arms are broken, leading to causes malfunctioning communication, modular robots can still work well. The reward setting is the same as Robosuite [295], and the cost design is used to prevent robots from crashing into unsafe areas.

Lift & Stack

The task is to grasp and lift a blue object from a table up to at least 0.1m of height. At the same time, to assure safety, we require that the robotic gripper avoids physical contact with the table while ensuring joint actions' smoothness, *i.e.*, keeps at least 0.02m (without smoothness constraints, see Equation (7.41)) or 0.001m (with smoothness constraints, see Equation (7.42) and Equation (7.43)) distance from it. For visualisation, see Figure 7.5,

⁵<https://github.com/chauncygu/Safe-Multi-Agent-Robosuite.git>

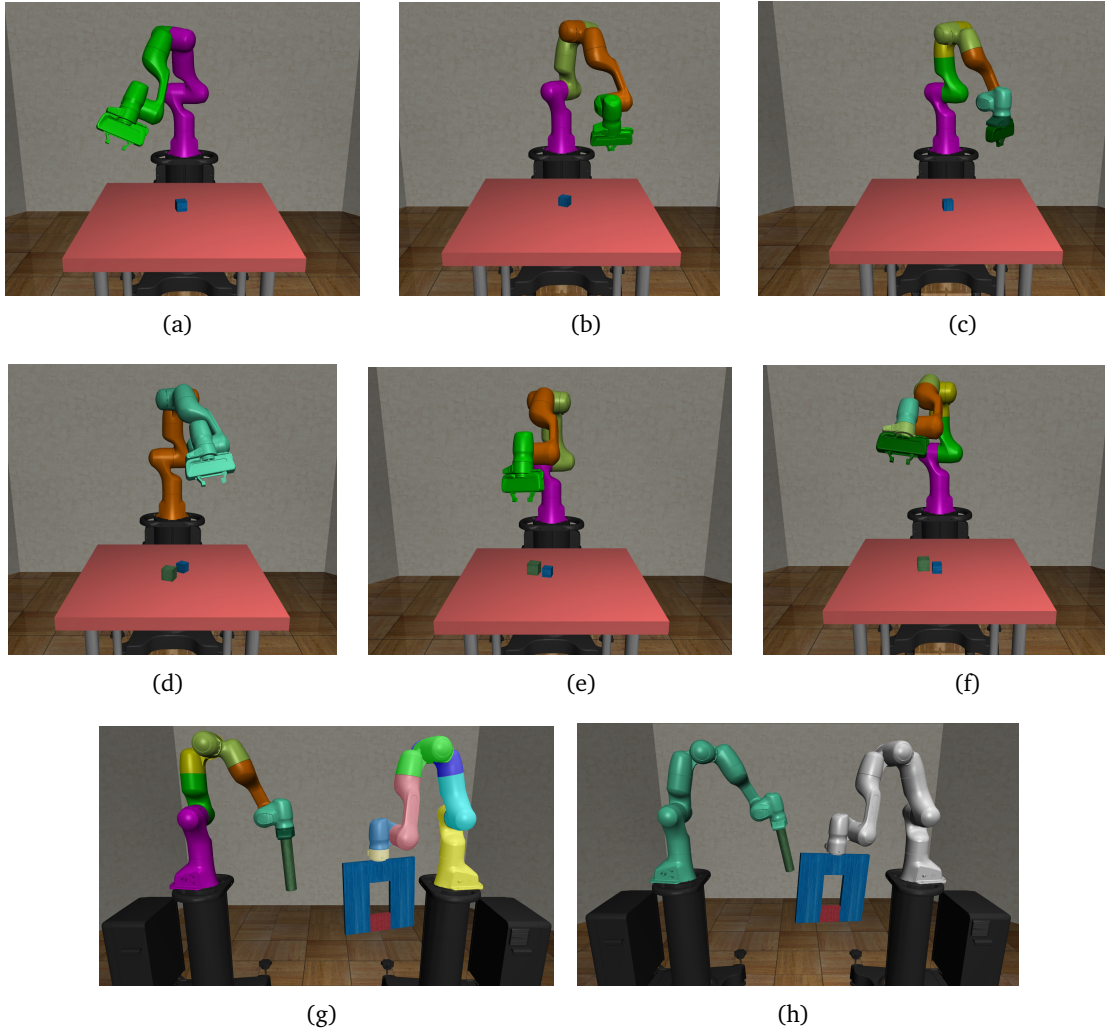


Figure 7.5: Example tasks in Safe MARobosuite Environment. (a): Safe 2x4-Lift, (b): Safe 4x2-Lift, (c): Safe 8x1-Lift, (d): Safe 2x4-Stack, (e): Safe 4x2-Stack, (f): Safe 8x1-Stack, (g): Safe 14x1-TwoArmPegInHole, (h): Safe 2x7-TwoArmPegInHole. Body parts of different colours of robots are controlled by different agents. Agents jointly learn to manipulate the robot, while avoiding crashing into unsafe red areas.

$$c_t = \begin{cases} 1.0, & \text{for } |\mathbf{p}_{\text{eef},t+1} - \mathbf{z}_{\text{obstacle}}| \leq 0.02, \\ 0, & \text{otherwise,} \end{cases} \quad (7.41)$$

$$c_t = \begin{cases} 1.0, & \text{for } |\mathbf{p}_{\text{eef},t+1} - \mathbf{z}_{\text{obstacle}}| \leq 0.001, \\ 0, & \text{otherwise,} \end{cases} \quad (7.42)$$

$$c'_t = \begin{cases} 1.0, & \text{for } |\mathbf{p}_{i,t+1} - \mathbf{p}_{i,t}| > 0.99, i \in [0, 7], \\ 0, & \text{otherwise,} \end{cases} \quad (7.43)$$

where $\mathbf{p}_{\text{eef},t+1}$ is the z -coordinate of the robot's end gripper, and $\mathbf{z}_{\text{obstacle}}$ is the z -coordinate of the obstacle; $\mathbf{p}_{i,t+1}$ is the action value of the agent i at time step $t + 1$.

TwoArmPegInHole

The agents learn to insert a peg into a hole of a rim. Additionally, the agents are required to keep the peg at at least 0.11m distance from a red part of the rim (see Equation (7.44) for cost and Figure 7.5 for visualisation).

$$c_t = \begin{cases} 1.0, & \text{for } p_{pd,t+1} \leq -0.11, \\ 0, & \text{otherwise,} \end{cases} \quad (7.44)$$

where $p_{pd,t+1}$ is the parallel distance between the robot's peg and robot's hole centre.

7.3.3 Safe Multi-Agent Isaac Gym

Safe Multi-Agent Isaac Gym (Safe MAIG)⁶ is a high-performance environment that uses GPU for trajectory sampling and logical computation based on Isaac Gym [164], see Figure 7.6. The computation speed of Safe MAIG is almost ten times that of Safe MAuJoCo and Safe MARobosuite on the same server, in the same task, using the same algorithm. The communication speed is also better than Safe MAMuJoCo and Safe MARobosuite. Specifically, we design the tasks for Safe MAIG in detail: ShadowHandOver (2x6, 6x2, 4x3, 12x1), ShadowHandCatchUnderarm (2x6, 6x2, 12x1), ShadowHandReOrientation (2x6, 6x2, 12x1), ShadowHandTwoCatchUnderarm (2x6, 6x2, 4x3, 12x1), ShadowHandCatchAbreast (2x6, 6x2, 4x3, 12x1), and ShadowHandOver2Underarm (2x6, 6x2, 4x3, 12x1). These tasks are visualized in Figure 7.6.

ShadowHandOver (2x6, 6x2, 4x3, 12x1)

The environment involves two hands at fixed positions. The first hand with an object must find a way to hand the item over to the second hand, and the second hand must learn how to grasp the item that is from the first hand, while one finger on the first hand has safety constraints (see Figure 7.6 (a)) over the range of motion of one of the fingers. Formally, the cost is given by

$$c_t = \begin{cases} 1.0, & \text{for } |F_{a4,t+1}| \geq 0.1, \\ 0, & \text{otherwise,} \end{cases}$$

where $F_{a4,t+1}$ is the first hand's fourth fingers's motion degree.

ShadowHandCatchUnderarm (2x6, 6x2, 4x3, 12x1)

This task is an extension of ShadowHandOver in which the hands can move freely, *i.e.*, they can translate/rotate their centres of masses within some region (see Figure 7.6 (b)). The safety constraints are as in the previous task.

ShadowHandReOrientation (2x6, 6x2, 4x3, 12x1)

In each of two hands there are two items. The goal of the agents is to rotate the two items around each other (see Figure 7.6 (c)). The safety constraints remain the same.

⁶<https://github.com/chauncygu/Safe-Multi-Agent-Isaac-Gym.git>

ShadowHandTwoCatchUnderarm (2x6, 6x2, 4x3, 12x1)

In each of two hands there is a item, and the item in the right hand will be handed to the left hand, and the left hand will hold two items well. The safety constraints are as in the previous task (see Figure 7.6 (d)).

ShadowHandCatchAbreast (2x6, 6x2, 4x3, 12x1)

This task is similar to ShadowHandCatchUnderarm, nonetheless, the two hands are placed side by side instead of in a position that is identical to ShadowHandCatchUnderarm (see Figure 7.6 (e)). The safety constraints remain the same.

ShadowHandOver2Underarm (2x6, 6x2, 4x3, 12x1)

Two hands are present in the environment in free postures, where they will grip the object cooperatively. The first hand holding the object must figure out a way to transfer it to the second hand, and the second hand must learn how to grip the object that came from the first hand. The object is thrown from the vertical hand to the palm-up hand in this scenario. The environment consists of the half ShadowHandCatchUnderarm and the half ShadowHandCatchOverarm (see Figure 7.6 (f)). The safety constraints remain the same.

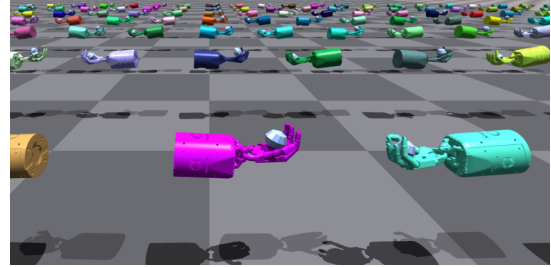
7.4 Summary of Safe Robot Reinforcement Learning Benchmarks

In this work, we have designed a comprehensive set of benchmarks to fairly evaluate the safety and reward performance of safe robot learning methods and to advance research in this field, e.g., *Safety-MuJoCo* Benchmarks, *Safety Multi-Objective MuJoCo* Benchmarks, *Safe Multi-Agent MuJoCo* Benchmarks, *Safe Multi-Agent Robosuite* Benchmarks, and *Safe Multi-Agent Isaac* Benchmarks. These benchmarks establish a standardized framework for assessing methodologies in safe robot learning, offering researchers a valuable tool for evaluating and comparing the safety and reward performance of various approaches in the field.

The introduction of these benchmarks represents a significant advancement for the safe RL community, providing researchers with a valuable tool to evaluate and compare the safety and reward performance of various approaches in the field.



(a) ShadowHandOver 2x6



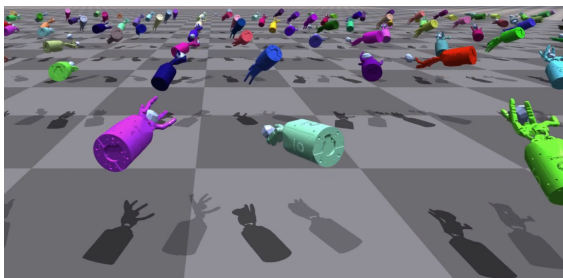
(b) ShadowHandCatchUnderarm 2x6



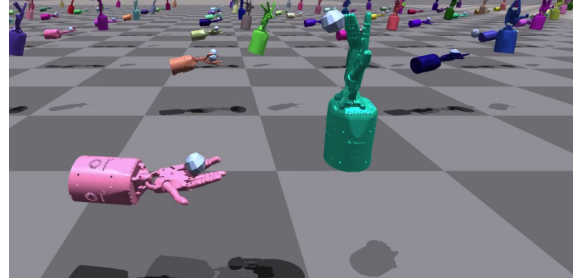
(c) ShadowHandReOrientation 4x3



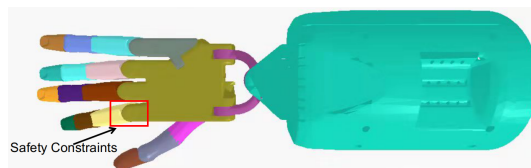
(d) ShadowHandTwoCatchUnderarm 2x3



(e) ShadowHandCatchAbreast 2x6



(f) ShadowHandOver2Underarm 2x6



(g) A hand showing safety constraints

Figure 7.6: Safe multi-agent Isaac Gym environments. Body parts of robots are controlled by different agents in each pair of hands. Agents jointly learn to manipulate the robot, while avoiding violating the safety constraints.

Conclusion and Outlook

8.1 Conclusion

In this dissertation, we aim to enhance the efficiency, multifunctionality, cooperativeness, and robustness of robot RL, specifically focusing on ensuring learning safety alongside the guarantee of monotonic improvement in reward performance. Our study is structured around a series of methods and contributions:

1. **Safe Balanced RL Method:** In this work, we introduce an approach to enhance the performance of safe RL by addressing the gradient conflicts that often arise between reward and safety optimization. This method is specifically designed to balance these two critical aspects, thereby improving the overall effectiveness of RL. Our approach achieves balanced optimization and demonstrates significant advantages over current SOTA baselines, particularly in safety and reward performance. This method offers a more effective and reliable solution for achieving near-optimal performance in safe RL applications.

2. **CR-MOPO for Multi-Objective RL:** To ensure learning safety in multi-objective RL, we introduce the CR-MOPO method. This approach is specifically designed to manage and optimize multiple objectives within safe RL frameworks, while ensuring safety throughout the learning process. This focus on both reward and safety performance represents a substantial advancement in safe multi-objective RL. The CR-MOPO method achieves a balanced performance across multiple objectives while maintaining a strong emphasis on learning safety. In comparison evaluations, it demonstrates superior performance relative to existing SOTA baselines.

3. **Safe Multi-Robot Learning:** To address the challenges associated with executing complex tasks in multi-robot RL systems, we have developed two safe multi-robot RL algorithms. These algorithms are designed to enable effective collaboration among multiple robots, allowing them to efficiently and safely perform complex tasks. A key feature of these algorithms is their strong emphasis on learning safety, ensuring that collaborative operations adhere to safety constraints throughout the learning process. A notable aspect of our approach is that it represents the first model-free method in safe multi-robot RL, providing a new framework for understanding and implementing collaborative robotic behaviors without relying on predefined models. In the comparison experiments, our method not only outperforms existing strong baselines in terms of reward performance but also ensures learning safety. This achievement highlights the effectiveness of our safe multi-robot RL algorithms and their potential for real-world applications.

4. **ROSCOM Algorithm for Uncertain Environments:** To address the challenges posed by uncertain environments in robot learning, we have developed the ROSCOM algorithm,

specifically tailored for safe RL applications across various settings. This algorithm is designed to prioritize safety while maintaining reward performance in uncertain environments. A key strength of ROSCOM lies in its ability to achieve near-zero safety violations, which is crucial in unpredictable and high-risk environments. Notably, our method improves safety performance by an impressive 99% compared to existing safe RL baselines, emphasizing its effectiveness in significantly reducing the risk of safety violations.

5. **Development of Safe RL Benchmarks:** To support and advance the broader research community, we have designed a set of benchmarks for safe RL. These benchmarks are designed for both single-robot and multi-robot learning research, offering a platform for fair and comprehensive evaluation of safe robot learning performance across diverse scenarios.

Each of these components represents a significant advancement in the field of safe robot RL, contributing to a more comprehensive understanding and application of safe RL in both theoretical and practical domains.

8.2 Outlook

Despite the development of various methods to ensure safety in robot learning, widespread implementation of RL in real-world applications still needs to be explored. In response, we highlight several current challenges and propose potential directions for future research in safe robot learning. These challenges and recommendations aim to guide further investigation to address gaps that currently limit the broad adoption of RL in practical settings. Advancing these areas is essential for moving the field toward more robust and reliable RL solutions.

8.2.1 Open Challenges

When deploying RL in real-world applications, numerous challenges arise during implementation. In this section, we outline key open challenges that need to be addressed in future research.

8.2.1.1 Human-Compatible Safe Reinforcement Learning

Modern safe RL algorithms rely on humans to state the objectives of safety. While humans understand the dangers, the potential risks are less noticed. Below we discuss two challenges in human preference statements and concerns on ethics and morality.

Human preference statement. Many challenges are posed as AI agents are frequently evaluated in terms of performance measures, such as human-stated rewards. On the one hand, while it is usually assumed that humans are acting in specifying their preference, such as by rewards or demonstrations, the consequence of humans mis-stating their objectives is commonly underestimated. Humans may maliciously or unintentionally mis-state their preference, leading the safe RL agent to perform unexpected implementations. One example is the Tay chatbot from Microsoft; prankster users falsify their demonstrations and train Tay to mix racist comments into its dialogue [75]. On the other hand, multiple humans might be involved in training one safe RL agent. Thus, agents have to learn to strike a balance between the widely different human preferences. Earlier attempt [97] considers one agent vs one human scenario. However, there are still many open questions, such as training robust agents against malicious users, personalizing assistance toward human preferences, etc.

Ethics and morality concerns. In modern society, the human interrelationship is built based on social or moral norms. While RL agents are deployed to the real world, they start

having impacts on each other, turning into a multi-agent system, in which norms act similarly in human society on agents. Therefore, the decisions made by agents always involve ethical issues. For example, social dilemmas will emerge from the relation between individual goals and the overall interests [108, 137]. The conflicts are produced when each agent aims to maximize its benefit. For another example, consider a *trolley problem* [78]. When the agent is faced with the choice of either harming multiple people on the current track or one person by diverting the train, what would you expect the safe agents to choose? Or more realistically, when the driving agent is about to bump into a lorry and it can swerve off the road to the left to save itself. But there is a bike on the left. How could the driving agent make decisions? A human driver's knee-jerk reaction might be swerving left to save itself. But the decision of the driving agent depends on its value systems. How to leverage the different value systems to enable safe agents to make ethical decisions is an open question.

8.2.1.2 Industrial Deployment Standards for Safe Reinforcement Learning

Although safe RL has been developed with a lot of well-understood methods and algorithms in recent years [81, 124, 154], to our knowledge, there is no RL deployment standard for industrial applications, including technical standards and morality standards, etc. More attention is required on deployment standard and the alignment between academia and industries. Applications include robotics, autonomous driving, recommendation systems, finance, *et al.* We should tailor specific deployment standards to specific applications.

Technical standards. In a technical standard, we need to think about how much efficiency RL can improve, how much time and money can be saved using RL methods, what environments can be handled with RL, how to design cost and reward functions considering the balance between RL reward, performance and safety, etc.

Law standards. Human-machine interaction needs to be considered in legal judgments. For example, when robots hurt humans due to programming errors using RL methods, we need to determine how responsibilities are divided, e.g., do programmers of robots need to take more responsibility, or do robot users need to take more responsibility?

8.2.1.3 Safety Guarantee Efficiently in Large Number of Agents' Environments

Since the decision-making space is incredibly large when the number of agents increases in a safe MARL setting, it is not easy to optimize the multi-agent policy to finish tasks safely [288]. Thus, efficiently guaranteeing safety in an environment with a large number of agents, e.g., 1 million agents, is still a challenging problem.

Theory of safe MARL. Theory and experiments of safe MARL for massive swarm robots should be taken into account in the future, e.g., the convergence problem still remains open for massive safe MARL methods without strong assumptions. Furthermore, it is essential to optimize sample complexity and stability within safe MARL environments. Particularly, greater emphasis should be placed on the following key points regarding the theory of safe MARL. (1). Credit assignment both in cost and reward. In cooperative, competitive, and mixed game settings, it is crucial to handle the trade-off between reward and safety performance, e.g., a policy should be searched to minimize each agent's cost value while improving reward. Furthermore, it is necessary to optimize the precise cost value for each agent, and consider each agent's cost credit. (2). Nonstability. In a multi-agent system, when an agent takes actions, which will influence other agents' actions and may make other agents get worse reward value or unsafe. (3). Scalability. In safe MARL settings, when the number of agents becomes large, such as one billion agents, it will be challenging for computation and hard to ensure agents' safety, since it is almost impossible to estimate each agent's Q value or V value simultaneously.

Multiple heterogeneous constraints. It still needs to be determined how multiple heterogeneous constraints should be handled in multi-agent systems for safe MARL. To our knowledge, almost no study has investigated multiple heterogeneous constraints for MARL. For example, when different agents encounter different heterogeneous constraints, we need to study how to balance different heterogeneous constraints to optimize different agents' policies for safety while improving reward values.

Carry out complex multi-agent tasks safely. For instance, swarm robots perform the encirclement task, and then rescue the hostages from the dangerous scene safely. Currently, most safe MARL algorithms could have some challenging issues while conducting complex tasks, e.g., time-consuming issues or convergence issues.

Robust MARL. One of the concerns in robust MARL settings: the problem of how to ensure zero cost in different tasks without tuning parameters using the same algorithms is still open [102]. Another one is: when we transfer the safe MARL simulation results to real-world applications, how to handle the mismatch between the nominal and real system is still unclear, since there may be the sensor noise [120] generated by fault sensors or disturbing sensor information transferred by adversary attacks.

8.2.2 Possible Directions for Future Research

Identifying potential research directions in the field of safe robot RL is crucial for advancing the understanding and application of RL. Key areas of focus could include:

1) Safe MARL with game theory. Addressing the aforementioned challenges by leveraging game theory constitutes a primary approach, as various games can be considered for real-world applications across different settings, including cooperative and competitive scenarios. Optimizing safety within extensive form games also proves beneficial for practical applications. For example, in a fencing competition, it is crucial to determine methods for ensuring that both agents successfully accomplish their objectives while maintaining safety.

2) Safe RL with information theory. Information theory can be instrumental in handling uncertain reward signals and cost estimation, as well as effectively addressing challenges in large-scale MARL environments. For instance, leveraging information coding theory allows for the construction of representations for various agent actions or reward signals, thus enhancing overall efficiency.

3) Safe MARL with human-brain theory and biology inspiration theory. Drawing inspiration from the laws of biology can contribute to the development of safe MARL algorithms. For instance, by studying the flight patterns of geese and comprehending how they form specific formations, we can enhance our understanding of safety mechanisms that ensure the well-being of each individual goose. This knowledge can then be applied to the design of safe MARL algorithms.

4) Learn safe and diverse behaviors from human feedback, like ChatGPT¹. During human-robot interaction, it is essential for robots to focus on learning safe and diverse behaviors from humans rather than discriminatory and illegal behaviors. We envision that a robot needs to adapt to different tasks and learn to satisfy different people's preferences after a robot can learn safe behaviors from one person. In such a scenario, a robot must learn to behave safely with multiple persons. For example, human feedback can be used as training data to boost the relationship between robots and humans and help guarantee exploration safety [79]. Moreover, behavior diversity is a critical factor to successful multi-agent systems [272], and diverse behaviors can be leveraged to search for safe policies for safe robot learning.

5) Human-robot interactions. Learning from interactions with non-expert users is essential

¹<https://openai.com/blog/chatgpt/>. Retrieval date: November 14, 2023.

for long-term safe RL. For example, an earlier attempt [97] considers learning the user's reward signal from human-machine cooperation. The prerequisite that robots work with humans safely and efficiently is mutualism. For successful collaboration with humans, robots must learn human preferences and comprehend human intentions and actions. By understanding what humans want and anticipate, robots can better adapt to their environment and receive enhanced support from their human counterparts. This mutual understanding fosters an effective and harmonious human-robot collaboration. Human behavior modeling [64] and realistic interactive modeling [296] can be the potential solutions for robots safely inheriting human preferences and learning more about humans.

List of Publications

1. **Gu, S.**, Yang, L., Du, Y., Chen, G., Walter, F., Wang, J., and Knoll, A. (2024). A Review of Safe Reinforcement Learning: Methods, Theories and Applications. *IEEE Transactions on Pattern Analysis and Machine Intelligence*.
2. **Gu, S.**, Shi, L., Ding, Y., Wierman, A., Knoll, A., Jin, M.. (2024). Enhancing Efficiency of Safe Reinforcement Learning via Sample Manipulation. *NeurIPS*.
3. **Gu, S.**, Liu, P., Kshirsagar, A., Chen, G., Peters, J., Knoll, A. (2024). ROSCOM: Robust Safe Reinforcement Learning on a Stochastic Constraint Manifolds. *IEEE Transactions on Automation Science and Engineering*.
4. **Gu, S.**, Huang, D., Wen, M., Chen, G., Knoll, A.. (2024). Safe Multi-Agent Learning with Soft Constrained Policy Optimization in Real Robot Control. *IEEE Transactions on Industrial Informatics*.
5. **Gu, S.**, Bilgehan S., Ding, Y., Wang, L., Lin, Q., Jin, M., Knoll, A. (2024). Balance Reward and Safety Optimization for Safe Reinforcement Learning: A Perspective of Gradient Manipulation. *AAAI*.
6. **Gu, S.**, Kuba, J., Chen, Y., Knoll, A., & Yang, Y. (2023). Safe Multi-Agent Reinforcement Learning for Multi-Robot Control. *Artificial Intelligence*.
7. **Gu, S.**, Kshirsagar, A., Du, Y., Chen, G., Peters, J., Knoll, A. (2023). Interactive Behaviors for Safe Robot-Reinforcement Learning. *Frontiers in Neurorobotics*.
8. Guan, J., **Gu, S.**, Li, Z., Hou, J., Yang, Y., Chen, G., & Jiang, C. (2023). UAC: Offline Reinforcement Learning with Uncertain Action Constraint. *IEEE Transactions on Cognitive and Developmental Systems*.
9. Hou, J., Chen, G., Li, Z., He, W., **Gu, S.**, Knoll, A., & Jiang, C. (2023). Hybrid Residual Multiexpert Reinforcement Learning for Spatial Scheduling of High-Density Parking Lots. *IEEE Transactions on Cybernetics*.
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11. Kuba, J. G., Wen, M., Meng, L., **Gu, S.**, Zhang, H., Mguni, D. H., Wang, J. & Yang, Y. (2021). Settling the Variance of Multi-Agent Policy Gradients. *NeurIPS*.
12. Zou, T., Chen, G., Li, Z., He, W., Qu, S., **Gu, S.**, & Knoll, A. (2021). KAM-Net: Keypoint-Aware and Keypoint-Matching Network for Vehicle Detection from 2D Point Cloud. *IEEE Transactions on Artificial Intelligence*.

13. Chen, G., Hou, J., Dong, J., Li, Z., **Gu, S.**, Zhang, B., ... & Knoll, A. (2020). Multi-Objective Scheduling Strategy with Genetic Algorithm and Time Enhanced A* Planning for Autonomous Parking Robotics in High-Density Unmanned Parking Lots. *IEEE/ASME Transactions on Mechatronics*.
14. Lu, F., Chen, G., Dong, J., Yuan, X., **Gu, S.**, & Knoll, A. (2020). Pole-based Localization for Autonomous Vehicles in Urban Scenarios Using Local Grid Map-based Method. *ICARM*.

List of Notations

$\mathcal{S}$	state space
$\mathcal{A}$	action space
P	transition dynamics
π	policy
$\mathbf{g}$	policy gradient
r	reward function
γ	discount factor
T	the number of steps
τ	trajectories
θ	neural network parameters
$\mathcal{M}$	Manifolds
VaR	Value at risk
$CVaR$	Conditional Value at Risk
C	cost value
c	safety bound on Euclidean space
δ	safety bound on Manifold space
η	safety probability on Manifold space
ϵ_{fi}	stochastic constraints on Manifold space
$\mathbb{R}$	real scalar number space
$\mathbb{R}^F$	F-dimensional real space
$\mathbb{R}^G$	G-dimensional real space
$\mathbb{R}^+$	positive real scalar number space
MDP	Markov Decision Process
$CMDP$	Constrained Markov Decision Process
ρ_0	the initial state distribution

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